Hume Implies Locke: The Fundamental Theorem of Property Theory

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Abstract

There is an invisible hand mechanism in the property system that underlies the invisible hand mechanism in the price system. In the life-cycle of property rights, initiation–transfers–termination, the "invisible judge" imputes the initial rights and terminal liabilities according to the public part of the life-cycle, the contractual transfers. If the legal system does not intervene, then the invisible judge *laissez-faire* imputes the termination of a property right to the last buyer and the initiation of a right to the first seller. When the legal system does intervene to hold a trial, it attempts to implement the principle of imputing *de jure* responsibility in accordance with *de facto* responsibility (the juridical version of the Lockean “fruits of one’s labor” principle). Hence the natural question is: under what conditions does the invisible judge satisfy the responsibility principle when no trial is held? Hume emphasized two basic conditions: that all transfers in property be voluntary contracts and that all contracts be fulfilled. The fundamental theorem for the invisible hand mechanism in the property system is that if Hume’s conditions are satisfied, then the invisible judge imputes in accordance with the Lockean responsibility principle. The paper mathematically formulates and proves the theorem using vector flows on graphs.

1 Introduction: The Product-Ownership Question

This paper is a mathematical introduction to a modern theory of property rights. To some extent the theory is new and to some extent it represents a reconstruction in modern terms of older and long-neglected ideas about property rights. The “property system” being modeled is an on-going market-based property system where appropriation applies to produced and consumed commodities (rivalrous goods).

The motivating question is:

"Who owns the new product produced using some means of production (e.g., land, buildings, machines)?"

The usual answer is:

Conventional answer: "The owner of the firm owns the product."

But what is this "firm" that can be owned? It is usually a corporation, the corporation that owns the, say, land, building, and machines used in the production process.

But it is easily shown that the "owner of the corporation" answer is not necessarily true since land, buildings, and machinery can all be rented or leased out so that the owner of those means of production is not the owner of the product produced using them. So how is product ownership determined in a market economy? The owner of the product produced is the legal party who bore the costs of production such as the inputs or intermediate goods used up in production as well as the services of the durable goods used in production. That party, which could be another corporation, is sometimes called the *residual claimant*. For instance, the residual claimant might have leased durable goods such as land, buildings, and machinery where the lease contract is interpreted as the sale of the services (e.g., machine-hours) of the durable goods.

Correct answer: "The residual claimant owns the product."

The important thing to note is that being the residual claimant is a *contractual role*, not a piece of property that can be owned. The owner of fixed assets might also play the residual claimant role, or if market conditions changed, then the only remunerative use of the fixed assets might be to lease them out to another party. In a market economy, renting out fixed assets does not violate any alleged property right ("ownership of the firm") of the owner of the assets. The ownership of the product is not determined by the ownership of some asset; it is determined by the contractual pattern of who hires what or whom.

2 The Invisible Hand Mechanism of Property Appropriation

Thus a market economy involves some mechanism by which product ownership is imputed to a certain contractual fact-pattern. This can be seen as an "invisible hand" mechanism in the property system. For the usual invisible hand mechanism in the price system, there is both a descriptive analysis (in terms of perfect competition) and a normative analysis (in terms of allocative efficiency in the sense of Pareto optimality). Then there is a fundamental theorem which says under certain circumstances (competitive equilibrium), the price system satisfies a certain norm (Pareto optimality).

Like price theory, property theory also has both a descriptive and normative side. And as in the price system, there is an invisible hand mechanism for the property system which imputes product ownership to the contractual arrangement of being the last owner of all the inputs to the production process so the party is the residual claimant. Our focus is on what might be taken as the fundamental theorem of property theory which plays a role similar to the fundamental theorem of price theory.[1]

Since Adam Smith, economic theory has worked to elucidate the invisible hand mechanism embodied in the price system of a market economy. The property system underlies the price system and accordingly the invisible hand part of the property system is concerned with simpler and more basic features. Given a set of legal contracts during a time period (whether they represent a price equilibrium or not), the basic contractual questions are realizing or fulfilling contracts with actual transfers of commodities (no breaches) and realizing only mutually agreed transfers so that all actual transfers of commodities are covered with voluntary contracts (no property externalities such as thefts). These principles have important roots in the Scottish Enlightenment, particularly in the work of Adam Smith's friend, David Hume. To these two principles of no breaches and no externalities, Hume added the respect for the possession of property to arrive at his three fundamental norms.

> We have now run over the three fundamental laws of nature, *that of the stability of possession, of its transference by consent*, and *of the performance of promises*. 'Tis on the strict observance of those three laws, that the peace and security of human society entirely depend; nor is there any possibility of establishing a good correspondence among men, where these are neglected. ([9, Book III, Part II, Section VI, p. 526]; cited in [8, p. 40])

There are two types of transfers involved: the voluntary contractual transfers of legal rights, herein called the *legal transfers* of the rights to commodities, and the transfers in the factual possession and control of commodities ("delivery"), herein called the *factual transfers*. In modern terms, we would think of two "spaces," an *ownership space* wherein legal rights are transferred by contract between parties and a *possession space*[2] wherein the objects of property, the commodities, are transferred between the parties. The two conditions "transference by consent" and "performance of promises" simply imply that the matching transfers are made in these two spaces.

When the factual and legal transfers do not match, there are two types of mismatches. A factual transfer not covered by a legal contract is a *property externality* or, in short, an *externality*. Hume's "transference by consent" is the *no-externality condition*. A legal contractual transfer of rights without the corresponding factual transfer or delivery of the property is a *breach of contract*. Hume's "performance of promises" is the *no-breach condition*. Together the conditions make "*Hume's matching conditions*" that factual transfers match legal transfers.[3]

What does the invisible hand do in the property system? A property system has to account for the whole life-cycle of property rights; how property rights are created, transferred, and terminated.

In a private property system, the contractual transfers of property between parties are the 'visible' or public part while the invisible hand deals with the creation and termination of property internal to the parties. The legal assignment of an initial property right may be referred to as the "appropriation" of the property right to some new commodity or asset. That is the product-ownership question broached above. That question also has a negative form. The inverse or opposite operation is the termination of a property right which may be conceptualized as the "appropriation of the liability" for the consumption, using-up, or destruction of some commodity or asset.[4] Hence the domain of the invisible hand mechanism is the

[1] See [1] for treatments of the basic theorems about the price system. See [5] or [6] for non-mathematical introductions to property theory.

[2] A transfer in possession space is not to be confused with movement in physical space. When a new buyer takes possession and control of a purchased house, then the house moves in possession space from one party to another even though the house does not move in physical space.

[3] That is the "Hume" in the title to the paper.

[4] The termination of rights was an original meaning of expropriation. "This word [expropriation] primarily denotes a voluntary surrender of rights or claims; the act of divesting oneself of that which was previously claimed as one's own, or renouncing it. In this sense, it is the opposite of 'appropriation'. A meaning has been attached to the term, imported from foreign jurisprudence, which makes it synonymous with the exercise of the power of eminent domain, ... " [2, p. 692, entry under "Expropriation"]. Since "expropriation" now has this acquired meaning, I will treat the "expropriation (termination) of rights to the assets $+X$" as the "appropriation of the liabilities $-X$."

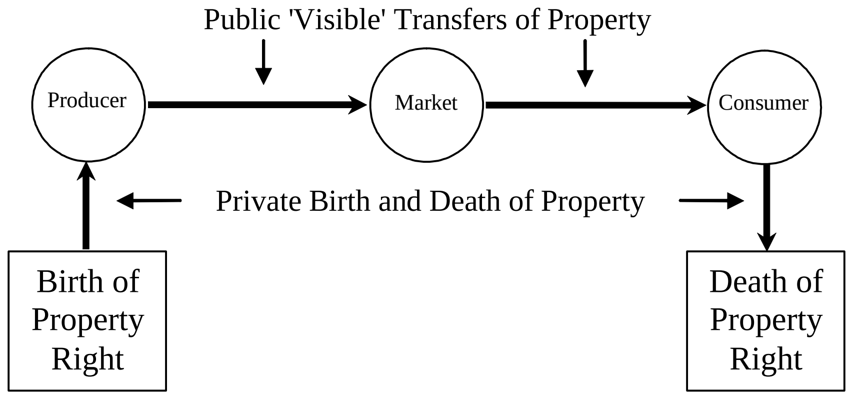

Figure 1: Life-cycle of a property right

mechanism for the appropriation of assets and liabilities.

To see the normative principle behind the invisible hand mechanism of appropriation, we take the principle applied by the legal system when it does intervene to, in effect, assign initial or terminal rights to assets. The prime example is a civil or criminal trial to assign the legal liability for property that has been destroyed. The underlying juridical norm is the:

Responsibility Principle:
assign the *de jure* or legal responsibility to the person or persons
who were actually *de facto* responsible for destroying the property.

Historically, this is often associated with John Locke's principle of people appropriating the fruits of their labor.[5]

The invisible hand mechanism for the legal assignment of initial and terminal rights comes into play when there is no explicit trial—when the visible hand of the legal authorities does not intervene and when it thus, in effect, renders the *laissez faire* judgment of "let it be." Using the Smithian metaphor, we might conceptualize "non-action" on the part of the legal authorities as the ruling of the "invisible judge" who always rules "let it be."

In the tradition of Ronald Coase [4], there has been an emphasis on a legal system defining clear property rights. Yet property rights are defined as much by the inaction of the legal system as by its actions. When sparks from a passing locomotive burn the crop growing in a farmer's field and the invisible judge rules "let it be" (i.e., the legal authorities for whatever reason allow no action), then at least the right to take that specific action was, in effect, established on the part of the railroad.

The two contracts of particular significance are the first and last transfer contracts in the life-cycle of a commodity. When a newly produced commodity is first sold and the invisible judge lets it be, then the initial property right was, in effect, assigned to the first seller. Conversely, when a purchased commodity is subsequently consumed, used up, or destroyed and the invisible judge lets it be, then the liability was, in effect, assigned to the last buyer.[6]

[5] That is the "Locke" in the title.

[6] A property damage suit might arise when one party destroys property X last purchased by another party. The invisible judge imputes the liability $-X$ to the last buyer but that party might go to court with a damage suit to get the legal system (e.g., a visible judge) to reassign the liability $-X$ to the *de facto* responsible party. If the suit was successful, the visible judge assigns the *de jure* responsibility to the party *de facto* responsible for destroying X and the material damage payment to the plaintiff would, in effect, turn the responsible party into the *ex post* "last buyer" In normal consumption, the consumer (responsible party) of X is the *ex ante* last buyer so the *laissez-faire* imputation of the liability $-X$ by the invisible judge is correct. The fundamental theorem gives those normal "equilibrium" conditions that imply the correctness of the market

Market Mechanism of Appropriation:
Initiation of right to X (appropriation of $+X$) is *laissez-faire* imputed to first seller of X.
Termination of right to X (appropriation of $-X$) is *laissez-faire* imputed to last buyer of X.

The most important and consequential application of the market mechanism of appropriation is to normal production activities. Abstractly considered, one legal party purchases (or already owns from past purchases or activities) all the "inputs" to be used up in the production process. When those inputs are used up and new products or "outputs" are produced, then the last buyer of the inputs is in a position to be the first seller of the outputs *unless* the legal authorities would intervene to overturn both sets of contracts. Hence when no such intervention takes places—as in normal production—then that one legal party in effect legally appropriates a bundle of legal rights and liabilities, the input liabilities and the output assets.

Our goal is to develop the fundamental theorem which shows that under Hume's no externality and no breach conditions, the invisible judge imputes according to the underlying juridical principle of responsibility that would be applied by a visible judge, i.e., Hume implies Locke.

We will informally describe some simple examples of the basic ideas before developing the mathematical treatment with vector flows on graphs.

3 Simple examples of the basic ideas

3.1 Divergence Principle

The simple underlying mathematical idea is called the "divergence principle." Support that an incompressible fluid such as water is flowing over a plane surface (it could be in rivers or streams or just flooding over the surface) and the surface contains fluid sources (like springs of water) and sinks (such as holes in the ground). Then draw a closed curve such as a circle around any region of the plane (which might include sources and sinks). Then the *divergence principle* is that, in any time period, the net flow of the fluid out of the boundary around the region (outflows minus inflows) is equal to the net sources of fluid within the region (flow out of sources minus flow into sinks), i.e.,

Divergence principle: Net flow out of region = flow out of sources − flow into sinks within region.

In the graph representation, the nodes or points in the graph are the sources and sinks, and the directed arcs or arrows between the nodes represent the flows (where a positive flow is in the direction of the arrow and a negative flow against that direction). In the property interpretation, the nodes are persons or legal parties, and the flows along the arcs are vectors of property rights (positive components going in the direction of the arrow and negative components flowing in the opposite direction). The represented flows are always for some time period. If a commodity is stored and held over to the next time period, then that is treated analytically as using up the time t commodity (i.e., going down a sink) and then the equal amount of the commodity is created in the $t+1$ time period (i.e., coming out of a source).

The basic idea behind the market mechanism of appropriation and the fundamental theorem is that by monitoring or measuring the flows across the boundary, i.e., the transfers of commodities between legal parties, then, by the divergence principle, one can make inferences about the net production and consumption of commodities by the parties.

3.2 Metering Systems

Consider the metered systems for delivering flows of water, natural gas, electricity, or other commodities to a household. The goal is to legally charge the household for the right amount of the metered quantity that the household uses up. Ordinarily, a household would not resell water, natural gas, or electricity, but the conceptual scheme must take account of such possibilities. If the household had solar or wind generators, then

mechanism of imputation.

it might transfer electricity back to the grid, so the goal of the metering system is to charge the household for their net consumption of electricity by metering the net flow of electricity to the household. The divergence principle could also be stated as: net flows into a region = Sinks − Sources.

But the system is more complicated since there are two kinds of flows that we have called "factual" and "legal." In a metering system, the legal flow into a household would be what is shown on the meter while the factual flow is the actual flow (meters might malfunction or be bypassed). When there is supposed to be both A and B in equal amounts, then there are two types of error: the "A and not B" error and the "B and not A" error. Clearly the (A) metered flow into a household is supposed to equal the (B) actual flow into the household.

The A and not B error would be when the meter shows a certain amount delivered (which is what the utility company will bill for) but that less was actually delivered. This is the "breach" type of error, a legal flow not matched by the corresponding factual flow. The B and not A error would occur when the meter was bypassed so that the actual flow into the household was not metered. This is the "externality" type of error, a factual flow not covered by the corresponding legal transfer. Hume's two conditions rule out these two types of error. The "performance of promises" rules out the breach error and the "transference by consent" rules out the externality error.

An additional layer of complication arises when we consider the nature of the "sources" and "sinks" in the household. A correctly functioning metering system (i.e., Hume's two conditions are satisfied) will charge the household for the actual net inflow of the metered quantity (electricity, water, gas, or whatever). But ideally the charge should be for what the household is responsible for using up. There are two ways the household's responsibility for using up the commodity might differ from the actual consumptive "sink" within the household. Indeed, these are just Hume's two conditions if we conceptualize the household's internal activity as a "trade with Nature." The analogue within a household of the voluntary contracts (i.e., legal transfers) is the responsible action of the household, and the analogue of the actual or factual transfers of commodities between parties is the actual using up or destruction of the commodity (the net sink) in the household's possession seen as a transfer away to Nature.

Thus the analogue of the no-externality condition would mean no destruction of the commodity that is not by the responsible action of the party, i.e., no accidental destruction as in the case of leakages. This can be ruled out by a no-accidents assumption (the simplest option which brackets accidents aside from the analysis) or the theorems can be correspondingly changed to allow for accidents.

The analogue of the no-breach condition says that any responsible action using up the commodity is realized by the actual transferring away of the commodity in the household's possession to Nature. Perhaps one can "sin in the mind alone" but that is not a concern of the property system trying to correctly impute the responsibility for consuming commodities. Or one might imagine a way to consume commodities not in one's possession (sort of an "action at a distance") but we rule this out on physical grounds. Responsibility implies causality and possession so "breaches" in trades with Nature are ruled out by a "no action at a distance" condition.

The simplest form of the fundamental theorem of property theory is the result for metered systems: under the assumption of no accidents (and no "action at a distance"), if there are no externalities (i.e., no actual net inflows that were unmetered) and no breaches (i.e., all metered net inflows were realized by actual inflows), then the imputation of responsibility for using up the commodity according to the meter is correct, i.e., the metered net inflow to the household equals what the household was responsible for using up (all during some given time period).

The fundamental theorem developed later is essentially the vector version of this simple result for metered systems where the meters for transfers are replaced by the legal transfers, i.e., voluntary contracts between parties recognized by the legal system.

3.3 The Stock-Flow Identity in Accounting

One of the simplest examples of the divergence principle is the stock-flow identity in accounting: ending stock − beginning stock = inflows − outflows (during the time period). Recall that ending stock left over to the next period are analytically treated as being used up (sink) and the beginning stock carried over from the prior period is treated as being created (source). Hence the stock-flow identity is just the "net-sinks = net-inflows" version of the divergence principle. Accounting typically keeps track of three terms in the identity and then uses the stock/flow identity to calculate the fourth.

In accounting, the legal versus factual distinction is the distinction between what is "booked" versus the actual quantities involved. We can then see the logic of the fundamental theorem operate in accounting systems. Suppose we start with a beginning booked stock level in a merchandise inventory (assumed equal to the physical beginning stock level) which could be zero and we record the booked inflows (purchases of merchandise items). Then there are two choices of method according to which of the two remaining items to determine (outflows or ending stock) and then the other is inferred or imputed by the equation. In the periodic inventory method, the ending stock is determined by periodic physical inventories which are booked and the outflow ("Cost of goods sold") is then determined by the equation (see any accounting text). In the perpetual inventory method, the outflows (inventory changes due to sales) are booked and the booked ending stock level is then inferred by the stock/flow equation (physical inventories are still taken but not as often).

In the perpetual inventory system for retail merchandise, the stock/flow equation is used as follows:

$$\text{Booked beginning stock } (= \text{actual beginning stock}) + \text{Booked purchases} - \text{Booked cost of goods sold} \\ = \text{Inferred ending stock on the books}$$

It is the perpetual inventory system that has the closest similarity to the market mechanism of appropriation. Since the legal system does not ordinarily do "physical inventories" of property holdings, the best analogy would be to see the perpetual inventory system as booking the change in stock according to the booked flows:

$$\text{Booked increase in stock (ending} - \text{beginning stock)} = \text{Booked purchases} - \text{Booked cost of goods sold.}$$

Like the market system, the inventory system can go wrong when there is a mismatch between the booked (legal) transactions and the physical (factual) transactions. An externality is a physical flow that is not booked such as a theft of inventory not booked as a sale. A breach is a booked transaction that is not fulfilled such as an undelivered purchase. The intent of a retail merchandise inventory is that the only changes in stock levels should be by purchases and sales. The analogue of an accident inside a household would be inventory spoilage within the firm, i.e., the unintended transfer away of inventory "to Nature" (in contrast to inventory theft which is the involuntary transfer to another party).

The "fundamental theorem" for the merchandise inventory system would then be: if there are no externalities (i.e., no unbooked physical inflows or outflows of goods) and no breaches (i.e., all physical inflows and outflows are booked), then, in the absence of any spoilage, the change in stocks (beginning − ending stocks) inferred by the booked flows (Booked purchases − Booked cost of goods sold) is correct. If the beginning stock was correct due to a physical inventory, then the inferred ending stock would be correct.

4 Graph Theoretical Preliminaries

4.1 Directed Graphs

A directed graph G is given by two sets A (arcs or edges) and N (nodes or vertices) and a function which assigns to each e_j in A an ordered pair $(n_i, n_{i'})$ in $N \times N$ where $i \neq i'$ which is interpreted as e_j being a directed arc from n_i to $n_{i'}$. There could be parallel arcs between the same two nodes but no loops at a node.[7] Each node will represent a legal person or party while the arcs represent property transfers (legal and factual) between the parties.

[7] The notation, terminology, and treatment of the topic will tend to follow [12].

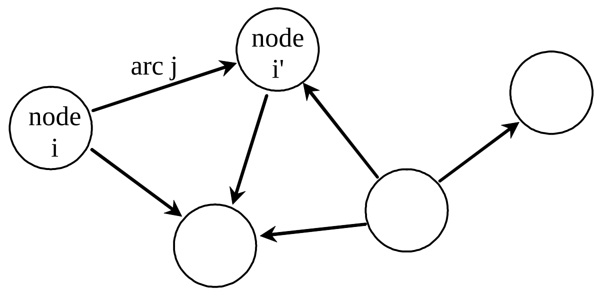

Figure 2: A Directed Graph

4.2 Incidence Matrix

Let the nodes and arcs be ordered so that $N = \{n_1, \ldots, n_m\}$ and $A = \{e_1, \ldots, e_n\}$. The node-arc incidence matrix is the $m \times n$ matrix $E = [e_{ij}]$ where $e_{ij} = 1$ if n_i is the initial node of arc e_j, where $e_{ij} = -1$ if n_i is the terminal node of arc e_j, and where $e_{ij} = 0$ otherwise. Thus each column (corresponding to an arc) has one $+1$ and one -1 with the rest of the column entries zero. Let $\mathbf{1}$ ($\mathbf{0}$ respectively) stand for the vector of all 1's (0's) with the dimensionality determined by the context. Since each column of $E = [e_{ij}]$ has one $+1$, one -1, and otherwise zeros, pre-multiplying by the $1 \times m$ row vector $\mathbf{1}$ will compute the column sums: $\mathbf{1}E = 0$.

4.3 Node and Arc Assignments

An assignment $x : A \to \mathbb{R}^c$ of real c-dimensional vectors to each arc is an arc assignment or flow and the value $x_j = x(e_j)$ is the vector flow through arc e_j. When $c = 1$, x is a scalar flow. In general, there are c different commodities. With vector-valued flows, x_j would be a property vector in the commodity space $\mathbb{R}^c$. The positive components of x_j represent the flows or transfers in the direction of the arc e_j and the negative components represents transfers in the opposite direction. An assignment $y : N \to \mathbb{R}^c$ of real c-vectors to each node is a node assignment where $y_i = y(n_i)$ would also be an c-vector in the vector-valued case. For the purpose of matrix computations, the arc or node assignments could be construed as $n \times c$ and $m \times c$ matrices.

4.4 Divergence Operator

Given a vector flow (vector-valued arc assignment) $x : A \to \mathbb{R}^c$, we compute at a node i,

$$\operatorname{div}(x)_i = \sum_{j=1}^{n} e_{ij} x_j$$

which gives the net outflow at node n_i from the flow x. Thinking of a fluid source at node i diverging to flow along the arcs, the node assignment div(x) is called the *divergence* [12, p. 11] of x (also *boundary* ∂x, e.g., [7, p. 27] with the opposite sign convention). If the vector $(3, 2, -3, 0)$ was transferred out of a node and $(2, -6, 0, 1)$ was transferred into the node, then the net out-transfer, net outflow, or divergence is:

$$(3, 2, -3, 0) - (2, -6, 0, 1) = (1, 8, -3, -1).$$

Using the $m \times n$ incidence matrix E and construing the flow $x : A \to \mathbb{R}^c$ as an $n \times c$ matrix, the divergence of x is a vector-valued node assignment $\operatorname{div}(x) : N \to \mathbb{R}^c$ which could be construed as the $m \times c$ matrix computed as:

$$Ex = \operatorname{div}(x).$$

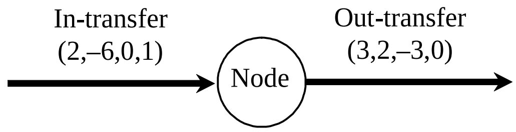

Figure 3: Net-Outflow = Divergence = $(1, 8, -3, -1)$

If we then add all the divergences of x over all nodes, then the flow on an arc will add in once positively and once negatively so the sum is the zero vector $\mathbf{0}$.

$$\text{Total divergence principle: For any flow } x,\ \sum_{i=1}^{m} \operatorname{div}(x)_i = \mathbf{0}.$$

Using the incidence matrix, the divergence is Ex and the sum of its elements is computed by pre-multiplying by $\mathbf{1}$:

$$\mathbf{1}(Ex) = (\mathbf{1}E)x = 0.$$

5 The Covering Ordering

To formulate the property theoretic concepts using vector flows on graphs, we will need an order relation so that we might say, for instance, that the factual transfers were "covered" by the legal transfers (no externalities). This "covering" ordering is, however, not the usual component-wise ordering on real vectors. The vectors assigned to an arc represent flows both in the direction of the arrow (positive components) and in the opposite direction (negative components). Yet the direction of the arrow is, in a sense, an arbitrary sign convention. A covering ordering (e.g., factual transfers are "covered" by contracts) should be stable under reversals in the direction of the arrow. For instance, if (apples, nuts) $= (2, -6)$ were the factual transfers from party A to B and $(3, -8)$ were the legal transfers, then we would say that the factual transfers were "covered" by the legal transfers. Each legal transfer exceeded the factual transfer in absolute value. The direction-independent statement is that the legal transfers were larger than or equal to the factual transfers in each direction. Yet $(3, -8)$ is not larger than or equal to $(2, -6)$ in the usual component-wise vector ordering. Hence we will define a new "covering" partial ordering on the real vectors to capture this property of the ordering being stable under reversal in the direction of the arrows.

A real vector can always be decomposed into the difference of two non-negative vectors. Given two vectors $X = (x_1, \ldots, x_c)$ and $Y = (y_1, \ldots, y_c)$ in $\mathbb{R}^c$, let $max(X, Y)$ be the vector with the maximum of x_i and y_i as its i^{th} component. The *positive part* of X is:

$$X^+ = max(X, \mathbf{0}),$$

the maximum of X and the zero vector. The *negative part* of X is:

$$X^- = max(-X, \mathbf{0}) = (-X)^+.$$

Both the positive and negative parts of X are non-negative (note the abuse of language in calling the non-negative vector X^- the "negative part" of X). Every vector X can be represented as the difference of the positive and negative parts, which is called the *Jordan decomposition*:

$$X = X^+ - X^-.$$

For instance, if $X = (2, -3)$, then $X^+ = (2, 0)$, $X^- = (0, 3)$, and $X = (2, 0) - (0, 3)$. Given two c-vectors X and Y, the standard vector inequality relation $Y \leq X$ holds between the vectors if the inequality holds component-wise, i.e., $Y_i \leq X_i$ for $i = 1, \ldots, c$. For instance, $0 \leq X^-$ and $0 \leq X^+$. The definitions of positive and negative parts and the inequality extend immediately to node and arc assignments by applying the definitions to all the vectors assigned to the nodes or arcs.

The *covering ordering* $\preceq$ is defined using the usual ordering on the positive and negative parts of the vectors:

$$Y \preceq X \text{ if } Y^- \leq X^- \text{ and } Y^+ \leq X^+ \text{ (read ``}X\text{ covers }Y\text{'' or ``}Y\text{ is covered by }X\text{'').}$$

The covering relation on real c-vectors is a *partial ordering* in the sense that it is reflexive (X covers X), transitive (if X covers Y and Y covers Z, then X covers Z), and anti-symmetric (if X covers Y and Y covers X, then $X = Y$). The zero vector is a minimal element in the covering ordering. Note that, as desired, the covering ordering is preserved under reversal of sign, i.e.,

$$Y \preceq X \text{ if and only if } -Y \preceq -X$$

whereas the usual inequality is reversed under sign reversal. The covering relation and the usual vector ordering agree on the positive orthant but differ elsewhere.[8] If $Y = (2, -6)$ and $X = (3, -8)$ then $Y \preceq X$ but $Y \not\leq X$. All these definitions and results extend immediately to arc and node assignments by applying them to each vector assigned to the arcs or nodes.

6 The Contractual Mechanism

There are two parts of a property transfer between parties; the legal transfer of rights embodied in a contract and the factual transfer or delivery of the commodities that fulfills the contract.

Let $LT : A \to \mathbb{R}^c$ represent the transfers in legal property rights between the parties which are assumed to be mutually voluntary contracts. Thus LT represents the legal transfers recognized by the property system during the time period under analysis.

Let $FT : A \to \mathbb{R}^c$ be an assignment of vectors to the arcs which represents the transfer in the *de facto* possession and control of the commodity vectors between the parties during the time period. Thus FT represents the factual transfers in "possession space" between the "control zones" of the parties.

The factual transfers not covered by voluntary contracts are called *property-externalities* or just *externalities*. When there are no externalities, then the factual transfers are covered by the legal transfers:

$$\text{No externalities assumption: } FT \preceq LT.$$

The legal contracts unfulfilled by factual transfers are called *breaches*. When there are no breaches then the legal transfers are covered by the factual transfers:

$$\text{No breaches assumption: } LT \preceq FT.$$

Together the no-externalities and no-breach conditions give the "equilibrium" or

$$\text{Hume's matching conditions: } LT = FT.$$

It plays the role for the fundamental theorem of the property mechanism that the competitive equilibrium condition plays for the fundamental theorem for the price mechanism.

In the price system, the law of supply and demand operates as a negative feedback mechanism to push competitive prices towards an equilibrium, i.e., excess supply leads to a lower price and excess demand leads

[8] An alternative to using the covering ordering would be to have only non-negative vectors assigned to arcs, to have arcs going in both directions between the nodes, and to use the usual component-wise ordering in the positive orthant.

to a higher price. In the property system, there is also a mechanism to enforce a matching between legal and factual transfers, a mechanism that is taken for granted in price theory. The mechanism is simply that either sort of mismatch between factual and legal transfers is sufficient occasion for a visible judge to intervene to overrule the laissez-faire imputations of the invisible judge and to impose a penalty. When one party's property is factually transferred to another party without any covering voluntary contract (e.g., a conversion or theft of property), then those are grounds for a legal intervention, e.g., a damage suit. When there is a legally recognized transfer of ownership but no fulfillment by the factual delivery of the property, then those are also grounds for a legal intervention, e.g., a suit for breach of contract.

The penalties applied to violations of the law do not simply have the function of a price. The idea is not simply to enforce material damages but to apply penalties such as punitive damages or criminal penalties to make the costs of breaches or externalities prohibitive. In theoretical optimization problems, constraints can be represented by infinite costs attached to the points that do not satisfy the constraints [11, p. 263]. Penalties function to enforce constraints, not to guide the allocation of resources. In computational algorithms, there are prices to guide allocation and there are penalties (which differ by orders of magnitude)—as in the penalty method or "Big M" method in linear programming [14, p. 112]—to enforce constraints. An everyday example is the orders-of-magnitude difference between the payment in a parking meter (an allocative function) and a parking fine for unmetered use of a parking space (enforcing a constraint), e.g., 25 cents in the meter versus a $25 fine.

While the difference between a price and a penalty is clear in mathematics, much of the law and economics literature seems not to appreciate the extra "undefinable kicker" [3, p. 78] in a penalty. That literature just treats a penalty weighed by the probability of capture and conviction as the (probabilistic) price of a law violation—an error that is then compounded in the notions of an "efficient theft" or an "efficient breach." The contractual mechanism attempts to use penalties to *eliminate* breaches and externalities (not to only have "efficient" ones).

7 Analysis of Appropriation

7.1 Appropriation is the boundary of contract

A legal system would determine a node assignment $LA : N \to \mathbb{R}^c$ (as in "legal appropriation") where the positive components at each node would represent the assets legally appropriated by the party during that period and the negative components would similarly represent liabilities legally appropriated by the party. In the language of imputation or assigning of legal responsibility, LA represents at each node the assets and liabilities legally imputed to that party during the time period.

The invisible hand mechanism of appropriation, in effect, makes an assignment to the nodes (parties) based on an arc assignment (legal transfers). Each arc assignment of transfers LT determines the node assignment $\operatorname{div}(LT)$ of net outflows at the nodes. The idea of the invisible hand mechanism was that in the chain of buying and selling of a commodity, there would be a first seller and a last buyer. If the legal authorities did not intervene, then we might figuratively say that the *invisible judge* imputed the initial ownership of the commodity to the first seller and the legal liability for using up the commodity (the "terminal" ownership) to the last buyer. Since the arc assignment LT represents the legal transfers or contracts, the positive components $\operatorname{div}(LT)^+$ of the divergence represent precisely the node assignment of first-sold commodities at each node. Any good both bought and then sold by a party would cancel out in the $\operatorname{div}(LT)$ at that node. If the legal authorities do not intervene, then those commodities would be *laissez faire* imputed to those parties by the invisible judge so they would be included in the positive components of LA, i.e.,

$$\operatorname{div}(LT)^+ \leq LA^+$$

Laissez-faire imputation of initial right to first seller.

Similarly the negative components $\operatorname{div}(LT)^-$ of the divergence represent the last-purchased commodities at the nodes, so without any legal intervention, those liabilities would be legally imputed by the invisible judge to those parties, i.e.,

$$\mathrm{div}(LT)^- \leq LA^-$$
Laissez-faire imputation of terminal right to last buyer.

Thus using the covering ordering, the operation of the market, *laissez faire*, or invisible hand mechanism of appropriation covers the first-sales/last-purchases of the parties:

$$\mathrm{div}(LT) \preceq LA.$$

There might be other methods of legally assigning assets and liabilities, e.g., when legal authorities intervene *ex post* in trials to, in effect, write or rewrite contracts or to assign private rights that are not transferred by contract. If we assume that only market appropriation is represented in the model, then whatever is legally appropriated is covered by the last-buyer/first-seller method, i.e., $LA \preceq \mathrm{div}(LT)$, then we have:

$$LA = \mathrm{div}(LT)$$
Invisible hand mechanism of appropriation.

Since the divergence of an arc assignment is also called its "boundary" (e.g., the first and last contracts are the boundary of a chain of contracts), the market or invisible hand mechanism of appropriation could be expressed by the slogan: "Appropriation is the boundary of contract."

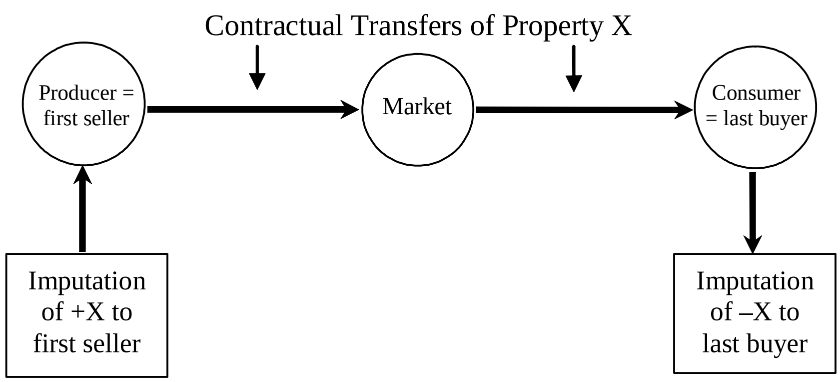

Figure 4: Appropriation is the boundary of contract

7.2 Responsible Actions and Internal Changes in Possession

So far, the analysis has looked at the contracts (legal transfers) between parties—and the legal imputations to the parties that are made, in effect, by the invisible judge. The factual transfers will be related to the legal transfers by the conditions of no externalities and no breaches. Now we turn to the internal factual activities of the parties. The simple model considered here is a one-period flow model so there is no separate treatment of stocks. In effect, any stock (e.g., inventory) carryover at the end of a period is treated as a flow used up or consumed at the end of the period. Accordingly, any stock carryover from the previous period is treated as being produced at the beginning of the period.

When people carry out intentional actions, then they are *de facto* responsible for the results of those actions. Let $RA : N \to \mathbb{R}^c$ (as in "responsible actions") be the node assignment representing in terms of net commodities the *de facto* responsible actions of the parties.[9] The sign conventions for RA were chosen so

[9] For instance, if party i performed the responsible actions L which used up the inputs K in the production of Q, often represented by a production function $Q = f(K, L)$, then representing only those commodities in the vector we would have $RA_i = (Q, -K)$. See the simple example developed below.

that the positive components represented commodities net produced (including being removed from stocks carried over from the prior period) while the negative components represented commodities net consumed or used up (or inventoried to stocks carried into the next period). Hence RA would be a node assignment with the positive (resp. negative) components at each node representing in net terms the assets (resp. liabilities) for which the party represented by the node is *de facto* responsible during the time period.

Not everything that happens to what is in one's possession happens deliberately. Let $\Delta P : N \rightarrow \mathbb{R}^c$ be the node assignment which at each node gives the net changes in possessions during the time period (whether the changes are intentional or accidental). As usual, the positive components represent net increases in possessions of the party at the node while negative components represents net decreases.

The assumptions about *de facto* responsible actions can now be expressed in the relationship between the node assignments RA and ΔP. The basic fact is that for a party to be *de facto* responsible for consuming or using up a commodity, then the commodity must have been in the *de facto* possession and control of the party (i.e., responsibility implies causality) and must have gone out of possession but not by transfer to another party. The consumer is the last possessor. There is no "action at a distance" (in possession space). Hence the fact that "the consumer is the last possessor" is expressed mathematically as:

$$RA^- \leq \Delta P^-$$

"Consumer is the last possessor."

Applied to production, the no-action-at-a-distance or responsibility-implies-causality principle implies that the commodities RA^+ that a party was *de facto* responsible for producing in net terms during the time period must be first *de facto* possessed by the party, i.e., must be included in ΔP^+. The producer is the first possessor. This is expressed as:

$$RA^+ \leq \Delta P^+$$

"Producer is the first possessor."

Using the covering relation, these combine to give us the:

$$RA \preceq \Delta P$$

No-action-at-a-distance (or responsibility-implies-causality) assumption.

This condition is stated as an explicit assumption but it might be better seen as simply a definitional consequence of the notions of *de facto* responsibility and possession. Responsibility implies causality so for a person to be *de facto* responsible for consuming or producing something, then it must have been at least under their causal control and possession to the extent necessary to be responsible for that effect. In that sense, responsibility implies possession, i.e., $RA \preceq \Delta P$. Analytically, this implies that when one person converts another's property to his or her own use without consent, then that would be analyzed as an illicit transfer (an externality), not as "action at a distance."

Using a simplistic dichotomy between deliberate actions and accidents, the gap between the net results of responsible actions RA ("fruits of the labor" of the parties) and the changes in possession ΔP would be the commodities represented in ΔP which were net creations or destructions that were not intended, namely net accidents (including "positive accidents" or windfalls). Accidents would be ruled out by assuming that whatever happened at the nodes (i.e., is in ΔP), happened deliberately (i.e., is in RA):

$$\Delta P \preceq RA$$

No-accidents assumption.

The taking of an unproduced natural good would be a windfall and would be ruled out of the model when the no accident assumption is made. The assumption of both no action at a distance and no accidents implies $\Delta P = RA$.

7.3 Divergence Principle: External Changes = Internal Changes

The arc assignment of factual transfers FT gives the transfers in the possession of the commodities between the parties during the time period. The net changes in possessions at the nodes resulting from the transfers is the divergence $\mathrm{div}(FT)$. That is external description of the changes in possession at the nodes. The internal description of the changes in possession at the nodes is ΔP. The relationship between the two can be illustrated by using the notion of an "augmented" graph [12, pp. 13-14]. Given any graph G, form a new graph G' which has one additional node d, the "distribution node," and an arc j_i from the new node to each of the original nodes. The vector flow from d to i through j_i represents the net created flow (sources minus sinks) internal to node i, e.g., the net increase in the possessions of the party i due to internal activities. In other words, the flow assigned to the arc j_i is ΔP_i.

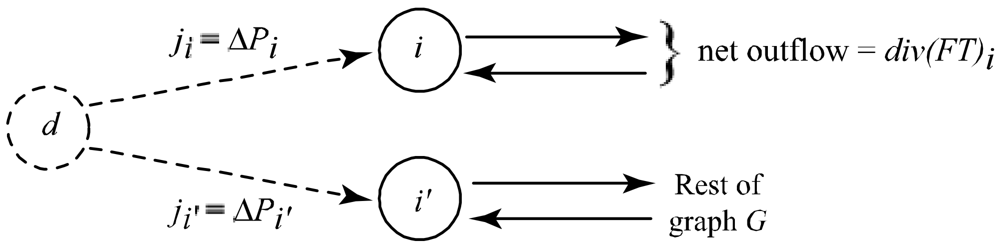

Figure 5: Augmented graph with distribution node

Economists in effect use this augmented graph when they think of production (minus consumption) as a "trade with Nature." The distribution node is "Nature" and an arc is oriented from Nature to each party. There are no other ways for possessions to change (i.e., trades with Nature or trades with other parties) so the flow conservation condition, $\mathrm{div}(FT)_i - \Delta P_i = 0$, at each node i equates the divergence of the factual transfers $\mathrm{div}(FT)_i$ with the internal changes in possessions ΔP_i for each party or node i:

$$\Delta P_i = \mathrm{div}(FT)_i$$

Continuity condition on factual transfers.

Flow conservation then automatically holds at the distribution node by the total divergence principle. This at first seems odd since it implies that the seemingly arbitrary node assignment ΔP must satisfy the total divergence condition (i.e., sum to the zero vector over all nodes in the original graph). But this follows from our convention of treating inventory carryovers between periods as being "consumed" at the end of the period and then "produced" at the beginning of the next period. If three bushels of apples were produced in a period but were still in inventory at the end of the period (e.g., were not sold or eaten), then they would be treated as being "consumed" in that period so they would not show up in ΔP representing net changes in possessions. Then they are "produced" again at the beginning of the next period and then, say, sold and transferred out to other parties. Then the three bushels would show up positively both in ΔP and in $\mathrm{div}(FT)$ for that period. Or consider the party who receives the transfer of three bushels of apples but does not resell or eat them during the period. Then the apples are nevertheless "consumed" in the inventory carryover at the end of the period so they would show up as a -3 bushels in the ΔP representing internal activities ("trades with Nature") as well as in $\mathrm{div}(FT)$ for the period. If they were actually consumed in the next period, then they would be both "produced" (inventory carryover) and consumed so would not show up in the net changes ΔP for that next period.

In the one-period flow models of the microeconomics textbooks, the whole life-cycle of property (production-transfers-consumption) is assumed to take place in each period so there is a constant (or no) carryover—and our one-period flow model is best interpreted in the same way.

This use of the augmented graph G' focused the continuity condition $\Delta P = \mathrm{div}(FT)$ which involved the factual transfers FT. We might also start with the legal transfers LT on the original graph G and ask what would be the "legal transfers with Nature" on the augmented graph G'? They are precisely the legal appropriations $LA = \mathrm{div}(LT)$ imputed by the *laissez-faire* mechanism of appropriation. Thus production and consumption are the factual transfers with Nature and the *laissez-faire* imputations are the as-if "legal transfers with Nature."

Returning to the factual flows, the name "continuity condition" is also motivated by fluid flow. Under that interpretation, ΔP represents the net source (sources minus sinks) of fluid at the nodes (like a spring coming out of the ground) while $\mathrm{div}(FT)$ represents the net outflow of fluid from the nodes through the arcs (pipes). The assumption that there is nowhere else for the fluid to go would then be expressed as the continuity condition: $\Delta P = \mathrm{div}(FT)$. Similar conditions in applied mathematics[10] express an "external = internal" equilibrium condition. In our case, $\mathrm{div}(FT)$ gives the external changes in possession at the nodes while ΔP gives the internal changes in possession.

The continuity condition can also be thought of as the local or point-wise version of the *divergence principle*. In a region of the plane bounded by a closed curve, the divergence theorem equates the net fluid flow across the boundary of the region with the sum of the net sources of fluid within the region [13, p. 187].[11] Taking the region as enclosing a single node, the divergence principle yields the continuity condition for that node.

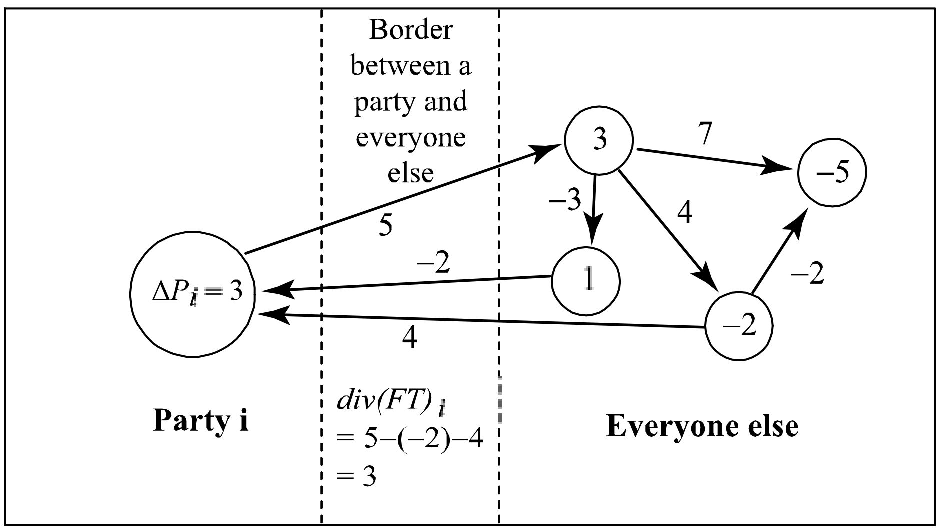

Figure 6: Illustration of Divergence Principle

Recalling the rules for flows along an arc (scalar flows in this case), the calculation in the next illustration is: outflow across border of party i is: $\mathrm{div}(FT)_i = 5 - (-2) - 4 = 3 = \Delta P_i$ = net source at party i [12, p. 55].

Finally, the juridical responsibility principle—if a party is *de facto* responsible for creating or using-up certain commodities, then the party should have the legal or *de jure* responsibility for those assets and liabilities—can now be stated in more formal terms using the covering ordering, i.e., that the legal appropriations cover the responsible actions:

$$\text{Responsibility principle: } RA \preceq LA.$$

[10] See the various interpretations of the $A^T y = f$ condition in [13].

[11] The one dimensional version is the Fundamental Theorem of Calculus. For example, consider a one-dimensional "tube" from point a to point b along the x-axis with the amount of the flow in tube at point x given by $F(x)$. At each point between a and b, there is a flow source of strength $F'(x) = dF/dx$ so by the divergence principle, the sum (integral) of all the sources (with positive F' representing an inflow) within the region or interval from a to b is equal to the difference between the ending and beginning flow in the tube: $\int_a^b F'(x)dx = F(b) - F(a)$.The simplest version of the divergence principle is the stock/flow identity in accounting which relates the net flows during a period to the change in stocks between the beginning and end of the period: outflow – inflow = beginning stock – ending stock.

8 Fundamental Theorem of Property Theory

With all the machinery in place, the fundamental theorem easily follows. The fundamental theorem connects the contractual mechanism (which enforces $LT = FT$) with the market mechanism of appropriation. We assume no action at a distance and no accidents and that only the market mechanism of appropriation is used to define legal imputations.

Theorem 1 (Fundamental theorem of property theory) *If Hume's matching conditions hold (no externalities and no breaches), then the market mechanism of appropriation satisfies the responsibility principle.*

Proof:
$RA =$ (by no accidents or responsibility implies causality)
$\Delta P =$ (by divergence principle)
$\mathrm{div}(FT) =$ (since $LT = FT$ by no breaches or externalities)
$\mathrm{div}(LT) =$ (by market mechanism of appropriation)
LA
so $RA = LA$.

This version of the fundamental theorem is easiest to explain since it does not explicitly involve the covering ordering. But the assumptions could be weakened to give a slightly stronger result by dropping the no accidents assumption.

Theorem 2 (Fundamental theorem allowing accidents) *Allowing accidents (including windfalls), Hume's matching conditions still imply the responsibility principle: $RA \preceq LA$.*

Proof:
$RA \preceq$ (by no actions at a distance)
$\Delta P =$ (by divergence principle)
$\mathrm{div}(FT) =$ (since $LT = FT$ by no breaches or externalities)
$\mathrm{div}(LT) =$ (by market mechanism of appropriation)
LA
so $RA \preceq LA$.

The "converse" theorem is a little harder mathematically and is only mentioned here for the sake of completeness.

Theorem 3 ("Converse") *Given de facto responsible actions RA that satisfy the quantity balance condition of the sum over all nodes being zero (total divergence principle), there exists a set of legal contracts LT such that fulfilling those contracts without externalities would give the actions RA as the divergence and would thus assign legal responsibility (LA) in accordance with the de facto responsibility actions RA.*

Proof: Assuming the graph is connected (otherwise repeat the argument on each component), the left nullspace $Null(E^T) = \{y \mid yE = \mathbf{0}\}$ (where E^T is the transpose of E) of its node-arc incidence matrix E is one-dimensional and consists of the scalar multiples of the $1 \times m$ row vector $\mathbf{1}$ [13, p. 74]. Taking RA to be a scalar assignment, the assumed total divergence condition is $\mathbf{1}RA = 0$ so RA is in the orthogonal complement of $Null(E^T)$ which is the column space of E. Hence RA is a linear combination of the columns of E so there is a y such that $Ey = RA$ where $Ey = \mathrm{div}(y)$. Thus $LT = y$ satisfies the theorem, i.e., $\mathrm{div}(LT) = RA$. The argument can be repeated for each commodity in the vector version of RA.

9 A Simple Example

An individual craftsman or producer buys one type of input and produces and sells an output so the vectors have three components: (dollars, outputs, inputs). Everyone else can be combined into one party, the market. The producer buys from the market K units of the input for \$$r$ each in cash, uses up those K units in the

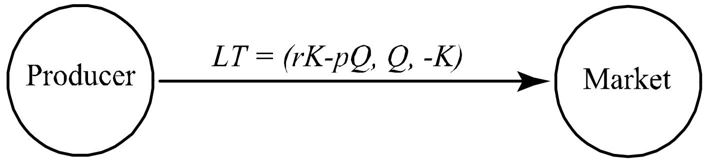

Figure 7: Legal Transfers between producer and market

production of Q units of output which are sold to the market for \$$p$ each. Hence the legal transfers are as follows.
The net outflow node assignment div(LT) at the producer node is:

$$\text{div}(LT)_{producer} = (rK - pQ, Q, -K)$$

which would also be the market imputation to the producer: $LA_{producer} = \text{div}(LT)_{producer}$. The producer is the last buyer of K so that property right is terminated in that party (i.e., the producer appropriates the liability $-K$). The producer is also the first seller of Q so that party is the initial owner of Q in the *laissez faire* imputation.

The money component is harder to interpret since it is not actually produced or consumed. But we are treating inventory (cash inventory in this case) carryovers at the beginning of the period as "production" and the carryovers at the end of the period as "consumption" With no other transactions (or accidental losses of cash), the ending stock of cash exceeds the beginning stock by $pQ - rK$ so that is the net consumption and thus the producer appropriates $-(pQ - rK) = rK - pQ$ as indicated by the formalism.

In the spirit of one-period models where there is no net change in the carryovers to and from the period, we might have the producer spend the valued-added $pQ - rK$ on buying from the market a fourth consumption good C which is consumed during the same period. Then the legal transfers would be as follows (adding the fourth component for the consumption good as in the next figure).

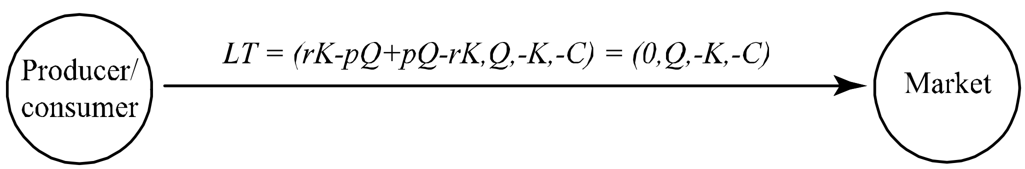

Figure 8: Legal Transfers between producer/consumer and market

Then the *laissez-faire* imputation to the producer/consumer is $(0, Q, -K, -C)$ with a clear interpretation. Assuming no externalities or breaches, these legal transfers were all fulfilled. With no accidents, the producer intentionally used up K in the production of Q and then consumed the good C purchased with the net proceeds so the imputation of $(0, Q, -K, -C)$ to the producer/consumer would be correct in terms of the responsibility principle—as per the fundamental theorem.

10 Concluding Remarks

The market mechanism of appropriation is based on the (divergence) principle that the external flows between a party and all the other parties must agree with the changes internal to a party. Under Hume's conditions, the *laissez-faire* mechanism of imputation based on contracts between parties will be correct in terms of the responsibility principle applied to each party's internal activities. Otherwise there are grounds for visible judges to intervene to explicitly apply the responsibility principle.

In philosophical terms, the responsibility principle is the juridical version of the principle of people appropriating the "fruits of their labor," a principle usually associated with John Locke.[12] The two conditions for the fundamental theorem to apply were Hume's conditions of "transference by consent" (factual transfers covered by legal transfers, i.e., no externalities) and the "performance of promises" (legal transfers covered by factual transfers, i.e., no breaches). Hence the shorthand form of the fundamental theorem is "Hume implies Locke"

In the contrapositive form—a violation of the Lockean principle implies a violation of Hume's conditions—the theorem states that if there was a misimputation by the invisible judge, then it would have to show up externally as a property externality or a breached contract. This is a property-theoretic refutation of Marx's charge that there could be exploitation in the "hidden abode of production" while the sphere of exchange "is in fact a very Eden of the innate rights of man" [10, p. 176].

Using the language of the Scottish Enlightenment, the theorem shows that in the "natural system of liberty," if the legal authorities can enforce Hume's no breach and no property-externality conditions, then the invisible hand mechanism of imputation—the invisible judge—will assign legal responsibility according to *de facto* responsibility, i.e., in accordance with the Lockean "fruits of one's labor" principle of property appropriation.

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[12] This is only intended as a historical tag, not an exercise in Locke exegesis. See [5] for an analysis of Locke's "fruits of your labor" principle.