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INTRODUCTION: PARETO VERSUS MARSHALL–PIGOU–KALDOR–HICKS

The Paretian revolution in normative economics established the possibility of defining efficiency (i.e., Pareto optimality, wherein no one can be made better off without making someone else worse off) without using interpersonal comparisons of utility or preferences. This treatment of efficiency is often seen as a weak form of a utilitarian or welfarist theory, that is, as a necessary condition for a maximum of "social welfare." But the notion of Pareto efficiency can also be seen as part of a rights-based approach[1] to normative economics that takes seriously the differences between persons and that accordingly eschews any given[2] social scalar ("social welfare") that morally ought to be maximized. Without any such scalar quantity to be maximized, the Paretian conditions are the necessary conditions for a vector maximization of the individual welfares.

The older Marshall–Pigou tradition in the economics of welfare was based on a fundamental distinction between the size and the distribution of the "social pie," for example, Pigou's "production" versus "distribution" of the "national dividend."[3] This social pie was *not* to be identified with overall welfare (e.g., Pigou's "economic welfare"), because the quantity of overall welfare could be affected by both the size and the distribution of the pie (e.g., Pigou's "national dividend"). The pie that economists would be professionally concerned with maximizing is an intermediate aggregate expressed in the measure of money and variously known as the national dividend (or product), net social benefits (e.g., in cost–benefit analysis), or social wealth (e.g., in the law-and-economics literature). An increase in *efficiency* was identified with an increase in the size of that social-wealth pie,

[1] See David Ellerman, 1992, *Property and Contract in Economics*, Cambridge, MA: Blackwell; David Ellerman, 2004, "The Market Mechanism of Appropriation," *Journal des Economistes et des Etudes Humaines*, 14, pp. 35–53.

[2] Persons can always come together, negotiate, and agree on certain common goals measured by a scalar quantity (e.g., profit in an economic enterprise), but those constructed goals are not ethically "given" independent of the common agreement.

[3] Arthur C. Pigou, 1960, *The Economics of Welfare*, 4th ed., London: Macmillan.

whereas the distribution of the pie was a question of *equity* outside the scientific bailiwick of economics.

Instead of taking the Paretian definition of efficiency in terms of vector maximization as an opportunity to explore nonwelfarist approaches to normative economics (e.g., rights-based theory), the economics profession has largely bridled at the "impracticality" of the definition. The rehabilitation of the Marshall–Pigou approach was inaugurated by the introduction of the Kaldor–Hicks (KH) principle[4] for a *potential* Pareto improvement (the winners in a proposed change could compensate the losers but do not necessarily do so) and by the modern treatment[5] of consumers' surplus. Kaldor was quite explicit about laying the groundwork to justify the older Marshall–Pigou way of thinking:

> This argument lends justification to the procedure, adopted by Professor Pigou in *The Economics of Welfare*, of dividing "welfare economics" into two parts: the first relating to production, and the second to distribution.[6]

The Marshall–Pigou tradition was thus modernized by Kaldor and Hicks, and the seemingly austere Paretian notion of efficiency[7] was broadened in the "Kaldor–Hicks (wealth maximization . . .) concept of efficiency."[8] Today any change that increases the "social wealth" according to the KH criterion is routinely interpreted as an "increase in efficiency," particularly in the law-and-economics literature, cost–benefit analysis, policy analysis,[9] and other parts of applied welfare economics.[10] In general, one could say that the closer economics is to being applied,

[4] See Nicholas Kaldor, 1939, "Welfare Propositions of Economics and Interpersonal Comparisons of Utility," *Economic Journal*, 49, pp. 549–552, and John R. Hicks, 1939, "The Foundations of Welfare Economics," *Economic Journal*, 49, pp. 696–712, for the original articles; and E. J. Mishan, 1964, *Welfare Economics: Five Introductory Essays*, New York: Random House, and John S. Chipman and John C. Moore, 1978, "The New Welfare Economics 1939–1974," *International Economic Review*, 19, pp. 547–584, for surveys of the KH criterion and later innovations. See Matthew D. Adler and Eric A. Posner, eds., 2001, *Cost–Benefit Analysis: Legal, Economic, and Philosophical Perspectives*, Chicago: University of Chicago Press, for a recent collection of viewpoints on cost–benefit analysis.

[5] The notion of consumers' surplus (Alfred Marshall, 1961, *Principles of Economics*, 9th ed., London: Macmillan) and the related notion of sellers' surplus are important tools in the Marshall–Pigou tradition, so Hicks's 1941 rehabilitation of consumer's surplus using utility-compensated demand curves ("The Rehabilitation of Consumers' Surplus," *Review of Economic Studies*, 8, pp. 108–116), and Robert D. Willig's 1976 justification of using Marshallian (uncompensated) demand curves as an approximation ("Consumer's Surplus without Apology," *American Economic Review*, 66, pp. 589–597), were important in reviving the thought patterns of that tradition.

[6] Kaldor, "Welfare Propositions," p. 551.

[7] Some authors assert that the KH principle will lead in the long run to a Pareto improvement. Aside from empirical problems, such a "defense" of the KH criterion is intellectually incoherent in that it attempts to reduce the KH condition to the Pareto or unanimity condition as if to admit that it was not an alternative after all. This chapter assumes that the KH condition is taken as a genuine alternative to the Pareto condition – as the foundation for economists to support some changes that will benefit some and hurt others on *efficiency* grounds.

[8] Richard Posner, 2001, "Cost–Benefit Analysis: Definition, Justification, and Comment on Conference Papers," in Adler and Posner, *Cost–Benefit Analysis*, p. 317.

[9] Edith Stokey and Richard J. Zeckhauser, 1978, *A Primer for Policy Analysis*, New York: W.W. Norton.

[10] See Richard E. Just, Darrel L. Hueth, and Andrew Schmitz, 1982, *Applied Welfare Economics and Public Policy*, Englewood Cliffs, NJ: Prentice-Hall.

the closer it comes to using the Marshall–Pigou approach with the modern KH refinements.

We will, however, show how the Marshall–Pigou–Kaldor–Hicks (MPKH) reasoning – based on the construction of a "pie" with the efficiency–equity parsing of changes in the pie's size and distribution – is fatally flawed.

I. NUMERAIRE ILLUSION AND SIMILAR SAME-YARDSTICK FALLACIES

Before considering the numeraire-illusion fallacy that vitiates the KH principle, it should be useful to consider a range of similar but simpler fallacies. The general fallacy involved here is the illusion that a statement is a substantive assertion when in fact it is only a tautological consequence of an arbitrary choice of numeraire, origin, or yardstick.

A "Proof" that Yardsticks Cannot Change

Suppose a yardstick is used to measure off a yard on a table. But is it really a yard? Perhaps the yardstick has expanded or contracted? Suppose that to check it, the distance is remeasured using the *same* yardstick, and sure enough (aside from negligible measurement error), the distance is indeed a yard. But this in fact gives no new information and simply reasserts the fact that the distance was originally measured by that same yardstick. Changes in a yardstick cannot be discovered by measurements using the same yardstick, and any conclusion of "no changes" based on such measurements would be illusory.

A "Proof" that Inflation Is Impossible

How much would a dollar buy in 1900? It would buy a dollar's worth of goods. How much would a dollar buy in 2000? It would again buy a dollar's worth of goods. Because a dollar buys the same "amount" of goods in 1900 and 2000, there has been no inflation between those two times. Because those two times were arbitrary, inflation is impossible.

What is wrong with this "proof"? Clearly the problem lies in using the same dollar measurement for what a dollar will buy at the two times. The seemingly substantive conclusion – "A dollar buys the same amount of goods at the two times" – is only a tautological restatement of the fact that the "amount of goods" is measured by what a dollar will buy.

A "Proof" that the Earth Does Not Move

Let $E(t)$ and $S(t)$ be respectively the coordinates of (the center of mass of) the earth and sun at time t when measured in geocentric coordinates. Then we check to see how the earth and sun move over the course of time. The investigation finds that indeed the sun's coordinates do change with the passage of time (in revolution around the earth) but that the earth's coordinates are constant. Therefore we can conclude that the earth does not move, because the sun does

indeed move in rotation around the earth. The Church is vindicated and Galileo refuted.

What is wrong with this "proof"? Clearly the illusion lies in the attempt to draw seemingly substantive conclusions about the movement of the earth from the mere choice of geocentric coordinates. Instead of being an empirical statement, "the earth does not move" is only a tautological consequence of choice of geocentric coordinates.

A "Proof" that the Marginal Utility of Income Is Constant

The marginal utility of income is the marginal rate of change of utility with respect to a change in the consumer's income. Let $U(Q)$ be the utility level that results from a consumer maximizing utility at given prices and income. Because any monotonic transformation of a utility function is equally acceptable as a utility function, we consider the money-metric utility function $E(P, U(Q))$, which is the minimum expenditure necessary to reach the level of utility $U(Q)$ at the given prices and income. Then we consider the marginal change in the money-metric utility $E(P, U(Q))$ with respect to a change in income. We find that the minimum expenditure necessary to reach the level of utility $U(Q)$ reached with, say, a dollar increase in income is exactly a dollar, so we conclude that the marginal utility of income is in fact constant (with value unity).

What is wrong with this "proof" that the marginal utility of income is constant? Instead of being an empirical statement about the marginal utility of income, it is only a mathematical consequence of the use of the money-metric utility function to measure the marginal utility of income:

> [T]he money-metric marginal utility *of income* is constant at unity. For how could it be otherwise? If you are measuring utility by money, it must remain constant with respect to money: a yardstick cannot change in terms of itself.[11]

Indeed, "a yardstick cannot change in terms of itself" is a good statement of the general same-yardstick fallacy.

A "Proof" that an Apple Has the Same Value to Any Consumer

If John had an apple, what would be its value to John? In terms of apples as numeraire, it would be worth one apple to John. If Mary had an apple, it would also be worth one apple to Mary. Hence an apple has the same value to John or Mary or to any consumer, so a transfer of apples between two people can never increase or decrease value. The argument can be restated in terms of any commodity (changing the numeraire accordingly), so any commodity has the same value to any consumer. Hence all transfers of commodity cannot increase value and are thus of no value.

[11] Paul A. Samuelson, 1979, "Complementarity: An Essay on the 40th Anniversary of the Hicks–Allen Revolution in Demand Theory," *Journal of Economic Literature*, 12, p. 1264.

What is wrong with this "proof"? Clearly the problem lies in measuring the value of an apple to a person and also using apples as the numeraire. The statement that the apple has the same value to John and to Mary is only a tautologous consequence of the choice of apples as the numeraire. The "same value" statement was only a numeraire illusion.

A "Proof" that Commodity Transfers Do Not Change Social Wealth

Although the preceding apple argument may seem obvious, the main point of this chapter is that the basic KH reasoning that *money* compensation payments (to turn a potential Pareto improvement into an actual Pareto improvement) do not change total social wealth *as measured in money* is only the same sort of tautologous restatement of the consequences of the choice of numeraire. If the total value, or pie, is measured in terms of the numeraire X (apples or money or any other commodity), then any transfers in X will only seem to be a redistribution of the same total pie and never as an increase or decrease in the size of the pie. Hence the parsing of the total Pareto improvement into the efficiency part that changes the size of the pie and the equity part that only redistributes some of the X without changing the X-measured size of the pie is only a consequence of the choice of the X numeraire. Change the numeraire to Y, and the same transfers of X will then (in general) change the size of the Y-measured pie, so the parsing is not numeraire-invariant.

Restrict attention to a Pareto improvement that exchanges an apple for some money (or apples for nuts), and the parsing of the total exchange into the efficiency part and the equity part using one commodity as numeraire will reverse itself when the other commodity is used as numeraire. Hence the policy recommendation of the non-numeraire transfer on efficiency grounds (because the numeraire transfers in the potential exchange are only a question of equity) will reverse itself with reversed numeraires. Lacking any serious argument that the social pie as measured by dollars, gold, silver, BTUs, apples, or nuts is the "true" or "normatively significant" social pie, such policy recommendations based only on the use of one particular numeraire are groundless.[12]

II. NUMERAIRE ILLUSION AND OTHER CRITIQUES OF THE KALDOR–HICKS PRINCIPLE

The key step in going from Paretian reasoning to the MPKH reasoning was the parsing of the total Pareto improvement into efficiency and equity parts using the

[12] There sometimes seems to be a type of *money mysticism* in the MPKH tradition that attributes some unspoken normative significance to using that good as numeraire. Monetized net benefits, as opposed to net benefits revalued using a different numeraire, are treated as if they represented social welfare, a mistake that Pigou was careful to avoid. This money mysticism is absent in the Paretian exchange perspective, which views money as one of many goods, albeit a particularly useful one, that may or may not be involved in mutually beneficial transactions. See John R. Hicks, 1975, "The Scope and Status of Welfare Economics," *Oxford Economic Papers*, 27, pp. 307–26, for an interesting juxtaposition of the catallactics (exchange) approach in its Lausanne and Austrian versions with the "production and distribution of the national product" approach of the Marshall–Pigou tradition.

criterion that the equity compensations (paid in the numeraire) did not change the size of the social pie (measured using the same numeraire). But this in only what we have called the *numeraire illusion*: changes in the size of a yardstick cannot be revealed by using that same yardstick. The illusion is that attributes of a description based on one numeraire (usually money[13] or, abstractly, "purchasing power") are misinterpreted as if they were numeraire-invariant attributes of the underlying situation being described.

It may be useful to differentiate explicitly this numeraire-illusion critique of the MPKH tradition from some previous criticisms. For instance, Scitovsky[14] pointed out certain problems in the KH criterion (e.g., the project and compensation might have such strong income effects that the KH criterion then recommended a return to the original state). This criticism shows that in certain theoretical cases, income effects can lead to anomalies that complicate the use of the KH criterion. To the purist, these anomalies may be seen as "nails in the coffin" of the KH principle. But in applied economics, the anomalies in very special cases did as little to slow the use of the KH principle as the majority voting paradox did to slow the use of majority voting. In any case, the critique based on the numeraire illusion has nothing to do the Scitovsky-type anomalies, and the critique applies to *all* uses of the KH principle (i.e., to the underlying logic), not just to special cases.[15]

Within the MPKH tradition, there is also some controversy about the relative importance of efficiency-versus-equity questions – as if the efficiency–equity parsing were a numeraire-invariant matter. Some applied economists, such as A. C. Harberger,[16] have argued that equity questions should be firmly set to one side so that professional interest can be focused solely on what are considered efficiency questions; "a dollar's a dollar for all that." Other welfare economists, such as Boadway and Bruce[17] and Blackorby and Donaldson,[18] do not accept the sharp separation of efficiency and equity questions (e.g., due to general-equilibrium effects); such questions are more intertwined and should be considered more jointly by economists. The criticism developed here shows the lack of invariance in the whole construction of the "social pie" and the parsing – intertwined or not – into efficiency and equity questions. It is independent of the question of how general-equilibrium considerations might intertwine the so-called efficiency and equity parts of the total change.

It should also be noted that the critique based on the numeraire illusion has nothing to do with the old idea of a dollar having a different social welfare impact

[13] For our purposes the numeraire is only the commodity used as the units in which benefits and costs are stated. The results do not depend on the numeraire having any of the other usual characteristics of money (e.g., store of value or medium of exchange).

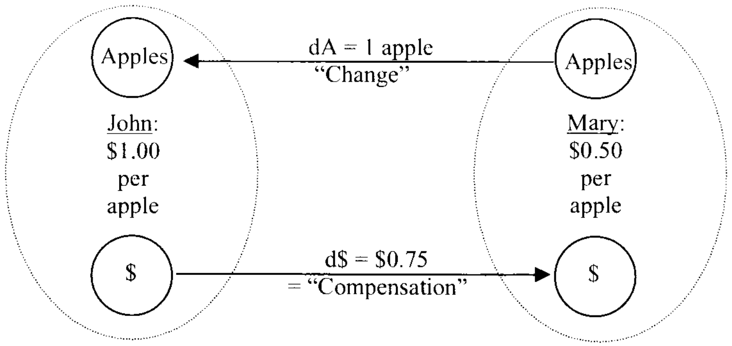
[14] Tibor Scitovsky, 1941, "A Note on Welfare Propositions in Economics," *Review of Economic Studies*, 9, pp. 77–88. Scitovsky's analysis generated a whole literature about such special cases, but it is not germane to our logical–methodological critique.

[15] Hence it is not just another nail in the coffin, but the last nail and final demise, of the KH principle.

[16] Arnold C. Harberger, 1971, "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay," *Journal of Economic Literature*, 9, pp. 785–97.

[17] Robin W. Boadway and Neil Bruce, 1984, *Welfare Economics*, Oxford: Basil Blackwell.

[18] Charles Blackorby and David Donaldson, 1990, "A Review Article: The Case against the Use of the Sum of Compensating Variations in Cost–Benefit Analysis," *Canadian Journal of Economics*, 23, pp. 471–94.



"Change" dA gives $0.50 = $1 – $0.50 = Δ$ increase in social $pie.
"Compensation" d$ gives $0 = $0.75 – $0.75 change in $pie.

Figure 6.1. The transfers described with $ as numeraire.

for the rich or poor, that is, the question of distributional weights in a supposed social welfare function. No notion of social welfare is used in this whole analysis and critique.

A. A Simple Generic Example

The MPKH reasoning is the basis for the maximization of "net social benefit" in cost–benefit analysis, as well as for the "social wealth" maximization at the foundation of the orthodox economic approach to law (the Chicago school of law and economics). For instance, consider the following pure example of numeraire illusion in cost–benefit analysis: "It should be emphasized that pure transfers of purchasing power from one household or firm to another per se should be typically attributed no value."[19]

In these contexts, it is not easy (though not impossible) to envisage a numeraire reversal, so the failure of numeraire invariance in hidden from normal view. But we are looking at the underlying economic logic of the MPKH tradition, and it can be applied to situations where numeraire inversions are trivial. Indeed, such examples are in law-and-economics textbooks themselves.

Consider the following simple but generic example from David Friedman's book *Law's Order*. Mary has an apple that she values at fifty cents, whereas John values an apple at one dollar. There might be a voluntary exchange where Mary sold the apple to John for, say, seventy-five cents. There are two changes in that Pareto improvement: the transfer of the apple from Mary to John, and the transfer of seventy-five cents from John to Mary (see Figure 6.1).

Let us apply social-wealth maximization reasoning to the transfer of the apple, using money as the numeraire. Because the apple was worth fifty cents to Mary and a dollar to John, social wealth would be increased by fifty cents by the apple

[19] Robin W. Boadway, 2000, *The Economic Evaluation of Projects*, Kingston, Canada: Queen's University, p. 30.

transfer from Mary to John. That is an increase in efficiency. The other change, the transfer of seventy-five cents from John to Mary, is a question of distribution or equity. Social wealth (measured in dollars and cents) would be unchanged by the mere transfer of seventy-five cents from one person to another:

> It would still be an improvement, and by the same amount, if John stole the apple – price zero – or it Mary lost it and John found it. Mary is fifty cents worse off, John is a dollar better off, net gain fifty cents. All of these represent the same efficient allocation of the apple: to John, who values it more than Mary. They differ in the associated distribution of income: how much money John and Mary each end up with.
>
> Since we are measuring value in dollars it is easy to confuse "gaining value" with "getting money." But consider our example. The total amount of money never changes; we are simply shifting it from one person to another. The total quantity of goods never changes either, since we are cutting off our analysis after John gets the apple but before he eats it. Yet total value increases by fifty cents. It increases because the same apple is worth more to John than to Mary. Shifting money around does not change total value. One dollar is worth the same number of dollars to everyone: one.[20]

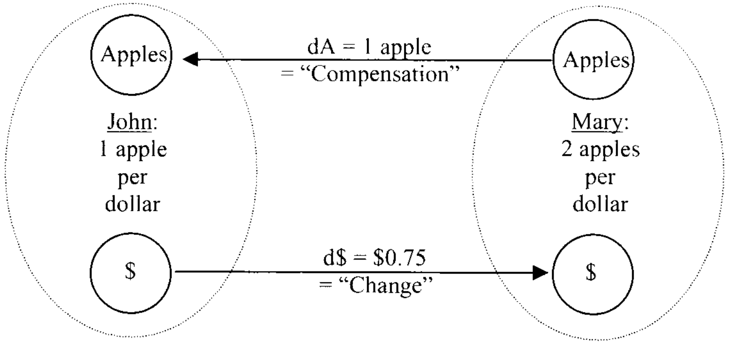
Now describe exactly the same situation but from an inverted perspective with a numeraire reversal from dollars to apples. Changing the numeraire does *not* mean the trivial conversion of the net benefits in one numeraire to another one at some fixed public price ratio; it means summing again the benefits and costs, using each person's marginal rates of substitution.

Mary was at a point where her marginal rate of substitution of dollars for apples was one-half, so her marginal rate of substitution of apples for dollars would be the reciprocal, namely, two apples per dollar. John's marginal rate of substitution of dollars for apples was one, and its reciprocal is also one. Now apply the reasoning of social-wealth maximization (measured in apples) to the proposed change of transferring seventy-five cents from John to Mary. The seventy-five cents is only worth three-fourths of an apple to John, whereas the seventy-five cents is worth three-halves apples ($2 \times 0.75 = 1.5$) to Mary. Hence the social pie (which is now an apple pie) is increased by three-fourths of an apple by the transfer of seventy-five cents from John to Mary. Hence the *money* transfer is the efficient change (the increase in social wealth).[21] Whether or not an apple is actually transferred from Mary to John is now a question of equity or redistribution that leaves the social (apple) pie unchanged. Paraphrasing Friedman's statement of the numeraire illusion, one apple is worth the same number of apples to everyone: one. An apple's an apple for all that (see Figure 6.2).

There has been no change in Mary's or John's preferences; exactly the same underlying situation is described, first using dollars as numeraire and then using apples as numeraire. Yet the results of the social-wealth maximization reasoning

[20] David D. Friedman, 2000, *Law's Order: What Economics Has to Do with Law and Why It Matters*, Princeton: Princeton University Press, p. 20.

[21] Taking any commodity as numeraire, the MPKH logic similarly recommends on "efficiency" grounds the transfer of money from those who relatively like to those who relatively dislike the commodity.



"Change" d$ gives 0.75 = 1.5 − 0.75 = Δapples increase in social apple pie.
"Compensation" dA gives 0 = 1 − 1 = no change in apple pie.

Figure 6.2. Same transfers with apples as numeraire.

(and the underlying MPKH logic) have changed completely between the two descriptions. The efficiency part and the equity part of the total change reversed themselves under the numeraire reversal. In contrast, it might be noted that the weaker catallactic conclusion that the two changes together constitute a mutually beneficial exchange (a Pareto improvement) is invariant under numeraire change (see Table 6.1).

The argument that "a dollar is worth the same number of dollars to everyone: one" pinpoints the problem that we have called the numeraire illusion. Transfers in whatever is taken as the numeraire will always seem to not change the size of the pie and thus to be merely distributive. Changes in a yardstick will never be revealed by *that* yardstick; one needs to use a different yardstick.[22]

If the mutually beneficial exchange had been Mary's apple in exchange for John's three-fourths of a pound of nuts (instead of three-fourths of a dollar), then in terms of some third commodity such as dollars we could say symmetrically that John values the apple more than Mary and that Mary values the three-fourths of a pound of nuts more than John. But by computing in the metric of one of the goods involved in the potential exchange, we are misled to the asymmetric conclusion that one part of the exchange increases the social pie whereas the other is mere redistribution of the social pie – an illusion that is exposed by changing the numeraire.

B. A Pollution Example

Law and economics (specifically, wealth maximization) applies the same logic of David Friedman's apple example to legal rules: "We now expand the analysis by

[22] The idea of a different yardstick is present in the idea of a *relative* price, the price of x in terms of y. The only price that has no information content is the "self-price" of the numeraire, one. The numeraire's price is like the blind spot in an eye, the one place where no information is conveyed. Given a (differentiable) function $y = f(x)$, the derivative dy/dx tells one something about the function, but the derivative $dy/dy = 1$ tells one nothing about the function.

Table 6.1. *Reversal of efficiency and equity parts under numeraire reversal*

Change	Normal description (money as numeraire)	Inverse description (apples as numeraire)
Increase in size of "social pie"	Transfer of apple from Mary to John	Transfer of seventy-five cents from John to Mary
Redistribution of "social pie"	Transfer of seventy-five cents from John to Mary	Transfer of apple from Mary to John

applying Marshall's approach not to a transaction (John buys Mary's apple) but to a legal rule."[23] Because so much of this approach to the economic analysis of law grew out of Ronald Coase's analysis of pollution,[24] such an example may be used to represent the methodology of law and economics.

Take the first numeraire y to be money, and take x to be the number of pollution permits.[25] Our points are independent of the question of polluter's rights or pollutee's rights, one that has received much attention in the literature on Coase's theorem. Hence we initially take a pollutee's-rights perspective and then later take the opposite viewpoint. In our first example, person 1 is the polluter, initially endowed with much money and few pollution rights, whereas person 2 is the pollutee, with the opposite relative endowments.

At the endowment point it might well be that there could some mutually voluntary exchanges of dy money for dx pollution permits between the polluter and pollutee. So far so good; it is a Pareto improvement due to voluntary exchanges in the market for pollution permits, with no need for the KH criterion or wealth-maximization reasoning.

The problem comes when, say, a legal–economic analyst of the Chicago school (or "the planner"[26] of cost–benefit analysis) uses the MPKH reasoning to analyze the transfer in pollution rights dx as an increase in social wealth, whereas the payments dy are seen as a merely redistributive transfer with no effect on the size of social wealth (aside perhaps from minor income effects). Economists and economics-savvy lawyers can recommend the efficiency change, the increase in social wealth due to the dx transfer, because the dy redistribution (e.g., polluters paying for pollution permits) is left aside as a noneconomic question (all *as if* the efficiency–equity parsing were an invariant attribute of the underlying legal situation rather than just a consequence of the choice of numeraire). Moreover, the merely redistributive dy transfer might be plagued by deadweight transaction costs that would actually reduce social wealth. Hence the most efficient outcome would be to make the social-wealth-increasing transfer dx to the polluter – in effect, to switch that part of the endowment to the polluter – and avoid any of the social-wealth losses due to the costs of the dy transaction. This would "mimic

[23] Friedman, *Law's Order*, p. 20.
[24] Ronald H. Coase, 1960, "The Problem of Social Cost," *Journal of Law and Economics*, 3, pp. 1–44.
[25] See, for example, the SO_2 permits analyzed in Denny Ellerman et al., 2000, *Markets for Clean Air: The U.S. Acid Rain Program*, New York: Cambridge University Press.
[26] Boadway and Bruce, *Welfare Economics*, p. 9.

the market" in terms of increasing social wealth, while avoiding the deadweight transaction costs.

All of these arguments and conclusions – representative of the Chicago school[27] – are vulnerable to the mere redescription of the situation by exchanging the numeraires. Gains and losses are now to be expressed in terms of the measuring rod of pollution rights (x), and the transfers can be analyzed from the viewpoint of the new social x pie. The money payment dy from the polluter to the pollutee increases social wealth (now measured in x), whereas the dx transfer of pollution rights merely redistributes x with no effect on total social wealth as measured in x.[28] One pollution permit is worth the same number of pollution permits to everyone: one. Economists can recommend the social-wealth-increasing transfer of the money dy from polluter to pollutee, whereas the question of transferring the pollution rights dx is best left aside as a noneconomic question. There might even be some deadweight costs in social wealth associated with the transfer of the pollution rights dx, so the most efficient outcome would then be to just reassign the money dy from the polluter to the pollutee. That would also mimic the market in terms of increasing social wealth, while at the same time avoiding the deadweight transaction costs.

The flaws in the MPKH reasoning have nothing to do with the Coase theorem controversy. The numeraire inversion analysis applies as in the preceding example if we start with the polluter's-rights principle. Now take person 1 to be the pollutee, relatively well endowed with money but few pollution rights – the latter being assigned to person 2, the polluter. Again we might expect at the endowment point that there could be some mutually beneficial voluntary exchange where the pollutee buys pollution rights dx from the polluter for the money dy. In the literature, this is sometimes viewed as the pollutee "bribing" the polluter to reduce pollution, or it could be seen as the purchase of amenity rights to the good of less pollution.

The problem comes in the MPKH reasoning that would analyze the transfer of dx amenity rights from the polluter to the pollutee as an increase in the social y pie while the payment dy had zero effect on that pie. If there are deadweight transactions costs involved in the otherwise redistributive dy payments, then the most efficient outcome is the uncompensated transfer of the amenity rights dx from the polluter to the pollutee. But these "policy recommendations" are easily reversed simply by redescribing the same situation with reversed numeraires.

With the amenity rights x taken as the numeraire, the payment dy from the pollutee to the polluter increases the size of the social pie (now measured in amenity rights), whereas the transfer of rights dx from polluter to pollutee is a "wash" because one pollution permit has the same value in terms of pollution permits to each party. And if there are transaction costs associated with transfer in amenity rights dx, then the MPKH–Chicago reasoning would conclude that the most efficient outcome is for the pollutee to make the bribe dy to the polluter but for the polluter to keep the pollution rights dx!

[27] Taking Chicago as the Mother Church in law and economics, we see here again a church mistaking a "does not move" statement (i.e., that social wealth "does not move" from transfers in the numeraire money or purchasing power) as a substantive assertion instead of a mere consequence of the choice of coordinate systems.

[28] See the appendix for a proof.

III. NUMERAIRE ILLUSION WITH CONSUMER AND SUPPLIER SURPLUSES

A. Summing Up So Far

There has been a long and rather exhausting debate in the law-and-economics literature about the KH principle, where the principle is often presented in the form of the wealth-maximization principle or, even, as the "efficiency norm."[29] In spite of the wide variety of arguments pro and con, the numeraire-illusion analysis seems to have escaped attention. Indeed, the numeraire-illusion analysis renders the debate rather moot. In an ultrasimple example such as the apple-and-money one, the wealth-maximization principle gives opposite results, depending on whether apples or money is taken as the numeraire. If money is the numeraire, then the apple transfer is wealth-increasing and the money transfer is not, but if apples are the numeraire, then the money transfer is wealth-increasing and the apple transfer is not. When the policy recommendation of the wealth-maximization principle reverses itself under a trivial redescription of exactly the same transfers, then the principle itself is rather incoherent – and the debate over it rather pointless.

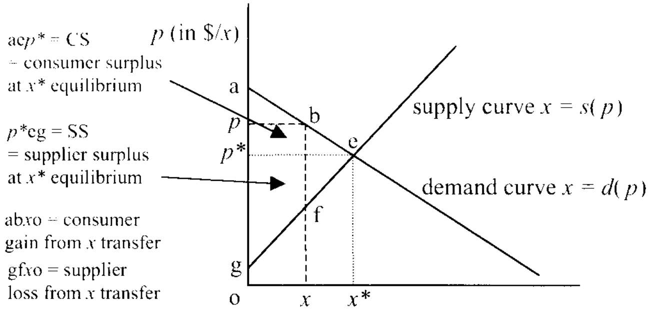
The vulnerability of the MPKH reasoning to numeraire change is not as obvious in the usual context of law and economics or cost–benefit analysis, but the underlying logic is the same. In the law-and-economics literature, the "apple transfer" might be some proposed change in the law. In cost–benefit analysis, the "apple transfer" would be some complex project under consideration, and the numeraire illusion is the statement that "pure transfers of funds among households, firms and governments should themselves have no effect on project benefits and costs."[30] When the "apple transfer" is a proposed legal change or a proposed project, the flaw in the MPKH reasoning is more hidden from view.[31] Perhaps that is why such a simple methodological error has persisted for so long. The problem in the underlying MPKH reasoning is easily exposed in Friedman's simple apples-and-dollars example[32] – and how could the reasoning suddenly become valid when

[29] See, for example, the collections of opposing viewpoints in "Symposium on Efficiency as a Legal Concern" in vol. 8 of *Hofstra Law Review* (1980); Mark Kuperberg and Charles Beitz, eds., 1983, *Law, Economics, and Philosophy*, Totowa, NJ: Rowman and Allanheld; or Adler and Posner, *Cost–Benefit Analysis*.

[30] Boadway, *Economic Evaluation of Projects*, p. 35.

[31] At the end of the appendix, the general result is stated that for complex multicommodity and multiperson transfers that together form a Pareto improvement, the MPKH reasoning will recommend all the transfers except the transfers in the numeraire, because the effects of the latter seem to vanish because of the numeraire illusion. Change the numeraire to one of the other commodities, and then the MPKH reasoning will recommend all the transfers (including those in the old numeraire) except the transfers in the new numeraire.

[32] A more realistic example would be a land reform program with positive net monetized benefits of land transfers from, say, the rich to the poor. Revaluating the benefits and costs in terms of land as numeraire (which we might assume is of a uniform grade) yields the result that the compensation



aeg = CS + SS = consumer + supplier surpluses = increase in $pie from x^* transfer.
0 = p^*ex^*o – p^*ex^*o = gain to x-supplier – loss to x-consumer from \$$p^*x^*$ transfer.

Figure 6.3. Standard supply-and-demand diagram.

apple transfers are replaced by more complex transactions in goods? Focusing on complex changes only fogs over the difficulties and does not resolve them. The "ostrich defense" – not looking at cases where the numeraire can be easily reversed – does not change the underlying logic (or the lack thereof).

B. The Standard Textbook Treatment

The numeraire illusion is hardly confined to the literature on cost–benefit analysis or the economic analysis of law. We will show how it arises in the standard textbook rendition of Marshall's consumer and supplier surpluses. There is a downward-sloping demand curve; the quantity of x demanded, x_d, is a function $x_d = d(p)$ of the price in dollars per unit x. And there is an upward-sloping supply curve; the quantity of x supplied is a function $x_s = s(p)$ of the price. Equilibrium occurs at a price p^* at which the quantities demanded and supplied are equal: $x^* = d(p^*) = s(p^*)$.

Leaving aside the fine-grained controversy about measuring the consumer and supplier surpluses as not germane to our analysis, the standard, or "naive," Marshallian definitions will be used. The total benefit to the consumer(s) in receiving x is measured in dollars by the area under the demand curve from 0 to x (see Figure 6.3). If px was paid out to receive x, then the net gain is the consumer's surplus. In a similar manner, the area under the supply curve from 0 to x represents the loss measured in dollars to the supplier(s) in giving up x. If px was received in return for x, then the net gain is the supplier's surplus.[33]

For the transfer of x from the supplier to the consumer, the total gain to the consumer is represented by the area ab$x$o, whereas the total cost to the supplier is the area gf$x$o. The difference is the total social surplus, represented by the area abfg. If the consumer paid px (the area pbxo) to receive x, then the difference is

payments to the rich are also a project that increases social value but that the actual land transfers have no impact on social value (as measured in land).

[33] We might also give MPKH their best case by assuming just one consumer and one supplier. Thus the apostrophe is before the *s* in *consumer's* and *supplier's* surpluses.

the consumer's surplus from the transaction measured in dollars and represented by ab$p$ in the diagram. If the supplier was paid px to give up x, then the difference between that revenue and the cost (gf$x$o) is the supplier's surplus from the transaction, represented by the area $p$bfg. The sum of the consumer's surplus and the supplier's surplus is again the total social surplus: (ab$x$o – pbxo) + (pbxo – gf$x$o) = ab$x$o – gf$x$o.

The most efficient amount of x to transfer is the one that maximizes the increase in the social $ pie, which is the equilibrium value x^*. Many textbooks still use this MPKH reasoning to "explain" the "efficiency" of the competitive equilibrium (in this market, the exchange of x^* in return for p^*x^* dollars). The Paretian explanation of efficiency (using up all the opportunities for mutually beneficial exchange) is usually also given as if the two accounts were equivalent.

But the difference between the two accounts of efficiency becomes clear as soon as we take the MPKH reasoning seriously enough to ask about the efficiency role of the p^*x^* payment. From the Paretian viewpoint, it is necessary to make the exchange *mutually* beneficial, a Pareto improvement, so the x^* transfer without the p^*x^* transfer does not pass Paretian muster.[34] But from the KH efficiency point of view, the payment p^*x^* is redistributive; it does not change the size of the social $ pie. Thus numeraire illusion arises in this standard textbook account by picturing the x^* transfer as generating by itself the consumer and supplier surpluses, whereas the p^*x^* transfer is only redistributive.

In spite of this reasoning being developed to facilitate economics giving "professional" or "scientific" advice to public policy, the reasoning in fact reverses itself after a mere redescription of exactly the same market with reversed numeraires.

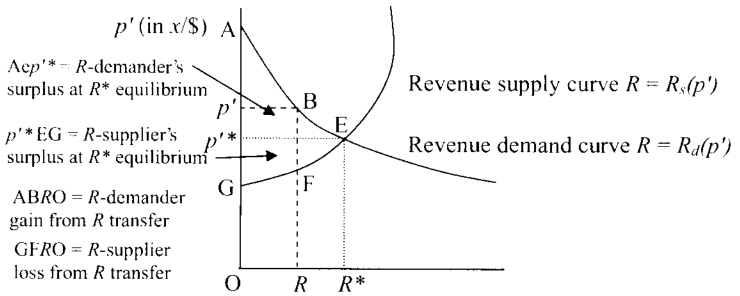
C. The Inverse Description

We now give an inverse description of the same market, reversing the roles of the commodity x and the revenue $R = px$. The supply curve provides the functional relationship giving the amount of x that is supplied if the revenue $R = s(p)p$ is paid for it. The seller of x goes to the market and demands money spent on x in exchange for x. Thus the x supplier is the R demander, and the reciprocal $p' = 1/p$ is the unit price of a dollar spent on x[35] in terms of x (where we may assume $0 < p < \infty$ and thus p' is in the same range). Thus the revenue demanded as a function of p' is $R_d(p') = R(1/p') = s(1/p')/p'$. This is the revenue (money spent on x) demand function in the redescribed market interpreting x supply as R demand:

$$\text{Revenue demand curve:} \quad R_d(p') = s(1/p')/p'.$$

[34] This highlights that the "mimic the market" rhetoric in Chicago-style law and economics selectively ignores the fact that market transactions involve payments.

[35] Intuitively, the commodity "dollars spent on x" could be thought of as money earmarked in a budget to be spent on x. The amount of this commodity supplied to or demanded from the market will depend on its price $p' = 1/p$ in terms of the numeraire x. Like an earmarked budget item, R units of this commodity can only be exchanged for $p'R$ units of x.



AEG = AEp'^* + p'^*EG = total increase in social x-pie from $\$R^* = \$p^*x^*$ transfer.
0 = p'^*ER^*O − p'^*ER^*O = gain to R-supplier − loss to R-demander of $p'^*R^* = x^*$ transfer.

Figure 6.4. Inverse description of market as supply and demand for R using x as numeraire.

For the illustrative case of a linear supply curve $x_s = cp - d$ (with c and d nonnegative), the revenue demand curve is the downward-sloping curve (in the positive R, p' quadrant:

$$\text{Revenue demand curve for a linear supply:} \quad R_d(p') = \frac{c}{p'^2} - \frac{d}{p'}.$$

The x demand curve gives the functional relationship between the amount of x that is demanded and the money or revenue $R = d(p)p$ supplied for it. We might think of the x demander as a money-spent-on-x or revenue supplier. The revenue supplied as a function of its unit price p' is thus

$$\text{Revenue supply curve:} \quad R_s(p') = d(1/p')/p'.$$

Because the revenue $R(p) = d(p)p$ is the product of a decreasing and an increasing function of p, it is not necessarily monotonic, and the revenue supply curve might be backward bending (in the positive R, p' quadrant). In the illustrative case of a linear demand curve $x_d = -ap + b$ (with a and b nonnegative), the revenue supply curve is indeed backward bending:

$$\text{Revenue supplied for a linear demand curve:} \quad R_s(p') = \frac{b}{p'} - \frac{a}{p'^2}.$$

An illustrative redescription of the x-and-R market is given in the diagram in Figure 6.4.

The equilibrium price p'^* in the redescribed market is the p' where

$$R_s(p') = d(1/p')/p' = s(1/p')/p' = R_d(p').$$

Multiplying through by p', we find that equilibrium occurs at the p' where $d(1/p') = d(p) = x_d = x_s = s(p) = s(1/p')$, which are the original equilibrium conditions for p^*. The quantities x demanded and supplied are equal at the price p^*, the equilibrium price in the market for x, so $p'^* = 1/p^*$. At p'^*, the equilibrium amount of the revenue R^* is $R_s(p'^*) = d(1/p'^*)/p'^* = d(p^*)p^* = x^*p^*$. The amount of x paid for R^* is the price times the quantity: $p'^*R^* = x^*p^*/p^* = x^*$. Thus the redescribed market gives exactly the same equilibrium – just looked at in the inverted way as the market for the supply and demand for money spent on x with the payments made in x.

Table 6.2. *Efficiency–equity reversal in standard competitive market analysis*

Kind of transfer	Normal description ($\$$ = numeraire)	Inverse description (x = numeraire)
Transfer that increased size of "social pie"	Commodity x^* from x supplier to x demander	Commodity $R^* = p^*x^*$ from R supplier (i.e., x demander) to R demander (x supplier)
Transfer that only redistributed "social pie"	Payment $R^* = p^*x^*$ from x demander to x supplier	Payment $p'^*R^* = x^*$ from R demander (i.e., x supplier) to R supplier (x demander)

It should not be surprising that the equilibrium properties of the model were unchanged by the mere redescription with reversed numeraire. However, the constructs of the MPKH reasoning change completely with the change in numeraire.

The area under the revenue demand curve (ABRO in the diagram) from 0 to R gives the total gain to the \$ demander (the x supplier), expressed in the numeraire x, from receiving R. The area under the revenue supply curve from 0 to R (GFRO) gives the total loss to the \$ supplier (the x demander) from giving up R. The difference gives the total social surplus, the increase in the social x pie, from the R transfer from the \$ supplier to the \$ demander. The transfer of the x payment for R, namely, $p'R = (1/p)px = x$, in the opposite direction is a mere redistribution of x that does not change the size of the social x pie.

The most efficient transfer of R is the amount that maximizes the increase in the social x pie – which is R^*. That transfer of $R^*(= p^*x^*)$ from the \$ supplier (x demander) to the \$ demander (x supplier) is the efficiency part of the transaction. The transfer of payment $p'^*R^* = (1/p^*)p^*x^* = x^*$ from the \$ demander to the \$ supplier does not affect the size of the social x pie, so it is the equity part of the total change – all according to the MPKH reasoning.

Thus in the textbook supply-and-demand competitive model, we have the ordinary description of the model with money as the numeraire (given in the textbooks), and we also have the inverse description with the commodity x as the numeraire (not given in textbooks). The underlying properties of the model (e.g., the equilibrium value of x^*, the equilibrium price ratio of \$ per x of p^*, and the equilibrium amount of money R^* transacted) are all the same under the redescription. But the MPKH parsing of the efficiency part and the equity part of the total change reverses under the redescription. The x^* transfer that is the efficient increase in the social \$ pie becomes a value-indifferent change in terms of the social x pie (i.e., the numeraire-illusion reasoning that "one unit of x has the same value in terms of x to everyone: one"). The $R^* = p^*x^*$ transfer of money, which was the value-neutral change in terms of the social \$ pie, becomes the efficient increase in the social x pie (see Table 6.2).

The numeraire-illusion analysis of the MPKH logic is not based on some esoteric special cases with little everyday relevance; the analysis applies to the simplest and most general textbook model of market equilibrium. Because the analysis in terms of efficiency (increased size of the social pie) reverses itself under the mere redescription of the same market with reversed numeraires, we see in the context of the standard textbook supply-and-demand model that recommendations based on MPKH efficiency reasoning are baseless. The MPKH shortcut to efficiency is a dead end.

What survives? The conclusion that is numeraire-invariant is that the mutual exchange of x^* for R^* is mutually voluntary, that is, that both transfers *together* are a Pareto-superior change. But this conclusion is based entirely on Paretian reasoning, and the MPKH efficiency–equity parsing and the wealth-maximization principle play no role.

FINAL REMARKS

One might ask: Where is economic reasoning misled by the numeraire illusion into making noninvariant conclusions? Where else have the "high priests" of economics habitually chosen to use "geocentric coordinates" and then "scientifically" drawn the conclusion that "the sun moves but the earth does not"? There is a whole research program to conduct an intellectual audit across economics to see where the numeraire illusion might lead to error as it did in the MPKH tradition of welfare economics.

Our focus here has been on Chicago-style (wealth maximization) law and economics, cost–benefit analysis, and other areas of applied welfare economics based on the MPKH reasoning. The common pattern is that a potential overall Pareto improvement is parsed into two parts: the proposed project or change, and the compensation of the losers that would make the total project cum compensation into a Pareto-superior change. Then the MPKH reasoning is used to represent the project by itself as an increase in the social pie measured by the money metric, and thus as something that can be recommended by economists on efficiency grounds. The compensation is represented as a redistribution of the social pie, a question of equity, not efficiency:

> The purpose of considering hypothetical redistributions is to try and separate the *efficiency* and *equity* aspects of the policy change under consideration. It is argued that whether or not the redistribution is actually carried out is an important but *separate* decision. The mere fact that is it possible to create potential Pareto improving redistribution possibilities is enough to rank one state above another on efficiency grounds.[36]

Richard Posner makes a similar point in the context of law and economics as well as cost–benefit analysis. He notes that KH efficiency leaves distributive considerations to one side:

> But to the extent that distributive justice can be shown to be the proper business of some other branch of government or policy instrument . . . , it is possible to set distributive considerations to one side and use the Kaldor–Hicks approach with a good conscience. This assumes . . . that efficiency in the Kaldor–Hicks sense – making the pie larger without worrying about how the relative size of the slices changes – is a social value.[37]

[36] Boadway and Bruce, *Welfare Economics*, p. 97.

Posner seems unaware that "making the pie larger" gives completely opposite results depending on whether it is the dollar pie or the apple pie (the \$ pie or the x pie). This pattern of reasoning – which assumes that the parsing of a proposed change into efficiency (size of pie) and equity (shares in pie) parts is a description-invariant property of the change – runs the length and breadth of the law-and-economics literature, and it is the warhorse of cost–benefit analysis and other parts of applied welfare economics.

As is clear from the numeraire reversals, there are simply no economic grounds to declare the project or change (e.g., x^* transfer) as an increase in efficiency and the compensation (e.g., $R^* = p^*x^*$ transfer) as a mere redistribution, rather than exactly the reverse. Both the project and the compensation are reallocations of resources that will each benefit some people and hurt others. The efficiency–equity analysis of the MPKH tradition does not provide valid economic grounds to claim that either partial change, which will benefit some and hurt others, can be recommended by itself on efficiency grounds.[38]

APPENDIX: THE ALGEBRA OF THE BASIC ARGUMENT

The controversies about measuring "consumers' surplus" or "aggregate willingness to pay" by integrating under Marshallian demand curves or Hicksian compensated demand curves are not germane to our point. Hence we will avoid those controversies by making the simple and basic point using differential changes (i.e., small changes at the margin) around a point prior to any integration over a path of finite changes.

There are two commodities X and Y involved in the changes. There are two *value systems* that give prices $P_1 > P_2 > 0$ of X in terms of Y that could be thought of as the marginal rates of substitution of Y for X of two different people, as the marginal rates of transformation of X into Y of two systems of production, or as resulting from any two different value systems in general. For example, the two prices might be the marginal rates of substitution of Y for X for persons 1 and 2:

$$P_i = MRS^i_{yx} = \frac{MU^i_X}{MU^i_Y} \quad \text{for } i = 1, 2$$

where $MRS^1_{yx} > MRS^2_{yx}$. Let P be any price between P_1 and P_2 that will function as a *public* rate of exchange between the two systems.

[37] Posner, "Cost–Benefit Analysis," pp. 318–319.

[38] Although beyond the scope of this chapter, it might be noted that this conclusion is congruent with the Wicksell–Buchanan perspective in political economy; see James M. Buchanan, 1999, *The Logical Foundations of Constitutional Liberty: The Collected Works of James M. Buchanan*, vol. 1, Indianapolis: Liberty Fund. Instead of using MPKH reasoning to supply an "efficiency" gloss to government planning decisions (e.g., cost–benefit analysis), it is the job of democratic politics to work out changes that are mutually voluntary on the part of all those whose rights are affected.

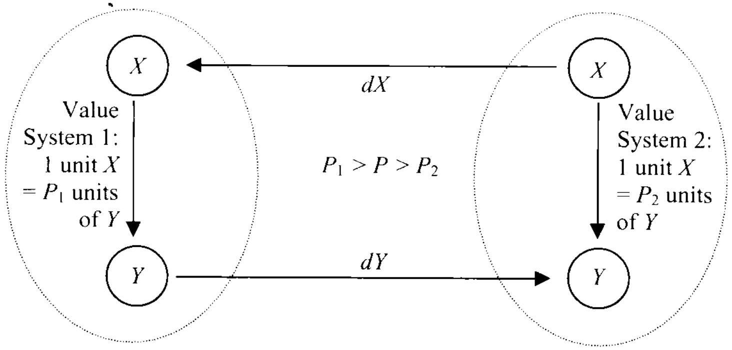


Figure 6.5. Two relative values for X in terms of Y.

We will further assume that these prices were determined by some prices of X and Y in terms of a third commodity Z. Let P_{1x} and P_{2x} be the prices of X in terms of Z that are *subjective*, or internal to the two systems, such that $P_{1x} > P_{2x}$, and let P_x be an intermediate public price of X in terms of Z. Similarly, let P_{1y} and P_{2y} be the prices of Y in terms of Z in the two systems such that $P_{1y} < P_{2y}$, and let P_y be an intermediate public price of Y in terms of Z. The previous prices of X in terms of Y are determined by the Z prices:

$$P_1 = P_{1x}/P_{1y} > P = P_x/P_y > P_2 = P_{2x}/P_2y.$$

The prices of Y in terms of X would be obtained by inverting the prices of X in terms of Y.

Thus we have three arrays of prices corresponding to the three different numeraires (X, Y, and Z). In the first description of transfers dX and dY between systems, or persons, 1 and 2, Y is the numeraire. Then we describe the same transfers with X as the numeraire. Both these descriptions will involve the numeraire illusion, because the numeraire is one of the commodities involved in the transfers being evaluated. The MPKH reasoning will apply to each of these cases but give opposite results. Then we evaluate the dX and dY transfers using a noninvolved commodity Z as the numeraire. Then no numeraire illusion arises, and the MPKH reasoning does not apply.

We start with the description using Y as the numeraire (see Figure 6.5).

If dX is transferred from where it has a lower value in 2 to where it has a higher value in 1, then the Y cost of taking dX out of 2 is P_2dX, whereas the gain from adding dX to 1 is P_1dX. Thus the increase in Y pie from the dX transfer is

$$\Delta Y = (P_1 - P_2)dX > 0.$$

Now suppose that $dY = PdX$ units of Y are transferred from 1 to 2. The cost to 1 is $dY = PdX$ units of Y, and the gain to 2 is dY units of Y, so the transfer of dY units of Y (or any other units of Y) yields no change in the size of the Y pie. The net change for 1 is $(P_1 - P)dX > 0$, and the net change for 2 is $(P - P_2)dX > 0$, so both 1 and 2 are better off, and the two positive slices add up to the Y pie:

$$(P_1 - P)dX + (P - P_2)dX = (P_1 - P_2)dX = \Delta Y.$$

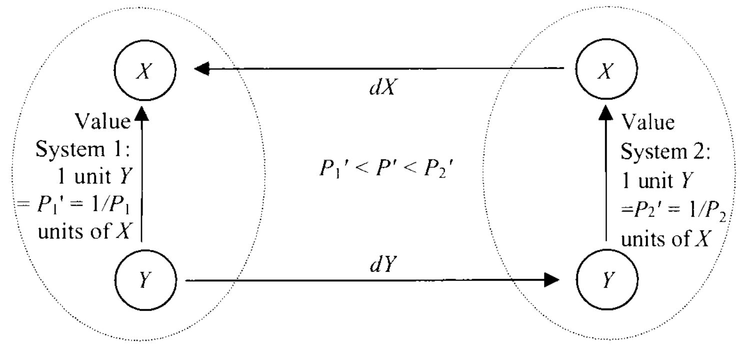


Figure 6.6. Two relative values for Y in terms of X.

The first term, $(P_1 - P)dX$, is the marginal version of X (consumer's surplus), and the second term, $(P - P_2)dX$, is the marginal version of X (supplier's surplus). The $dY = PdX$ transfers change the distribution of the Y pie between 1 and 2, but do not affect the size of the pie.

So far, this is just mathematics. Then the MPKH reasoning is misled by the numeraire illusion involved in measuring the effect of the dY transfer in terms of Y to conclude that the dY transfer added no value or wealth; it was only a redistribution. The value increase was all in the dX transfer, so it can be recommended on grounds of efficiency while the dY transfer can be treated separately as a question of equity.

But this asymmetric treatment of the dX and dY transfers is only a consequence of the asymmetric choice of one of the involved commodities as numeraire to evaluate the transfers. Reverse the choice of numeraires, and the conclusions will be reversed. Taking X as the numeraire, $P_1' = 1/P_1$ is the price of a unit of Y in units of X in system 1, and $P_2' = 1/P_2$ is the price of a unit of Y in terms of X in 2 (see Figure 6.6).

We now evaluate the results the same dY transfer from 1 to 2. The loss to 1 is $P_1'dY$, and the gain to 2 is $P_2'dY$, so, noting that $P_2' > P_1'$, we have the increase in the X pie from the dY transfer as

$$\Delta X = (P_2' - P_1')dY > 0.$$

Now taking $P' = 1/P$, we find that $P'dY = P'PdX = dX$ is the same dX units of X transferred from 2 to 1. The cost to 2 is dX units of X and the gain to 1 is dX units of X, so the transfer in dX units of X (or any other units of X) yields no change in the size of the X pie (i.e., the self-measuring yardstick records no change). But there is a change in the distribution of the pie. The net changes for 2 and 1 are respectively

$$(P_2' - P')dY > 0 \quad \text{and} \quad (P' - P_1')dY > 0.$$

so both 1 and 2 are better off, and the two positive slices sum to the X pie:

$$(P_2' - P')dY + (P' - P_1')dY = (P_2' - P_1')dY = \Delta X.$$

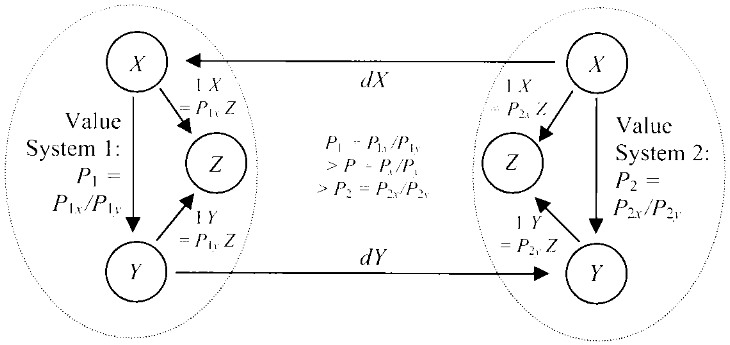


Figure 6.7. Two values for Y and for X in terms of Z.

The term $(P_2' - P')dY$ is the marginal Y consumer's surplus, and the second term, $(P' - P_1')dY$, is the marginal Y supplier's surplus.

These are the exact same underlying changes: the transfer of dX from 2 to 1 and the transfer of dY from 1 to 2. But the MPKH reasoning now yields the reverse conclusions. The dY transfer accounts for all the increase in the size of the X pie, so it can be recommended on efficiency grounds. The dX transfer merely redistributes the X pie, so that transfer can be treated as a separate question of equity.

One escapes the numeraire illusion only by evaluating the transfers in terms of some third commodity Z not involved in the transfers. We use the array of prices in terms of Z as assumed (see Figure 6.7).

The exchange of dX and $dY = PdX$ is equal-valued at the P_x and P_y prices, because $P_x dX = P_y dY$. But at the internal, or subjective, values in the two systems, the change in Z value from the dX transfer is $(P_{1x} - P_{2x})dX > 0$, and the change in Z from the dY transfer is $(P_{2y} - P_{1y})dY > 0$. The sum of the two increases is the total increase in the Z pie from the dX and dY transfers:

$$\Delta Z = (P_{1x} - P_{2x})dX + (P_{2y} - P_{1y})dY.$$

Because the exchange of dX and dY is made at the intermediate prices P_x and P_y, where

$$P_{1x} > P_x > P_{2x} \quad \text{and} \quad P_{1y} < P_y < P_{2y},$$

we can compute the surpluses in each system in terms of Z. In system 1, the gain from receiving dX is $P_{1x}dX$, while the cost of losing dY is $P_{1y}dY$, so the net benefit to 1 is

$$\begin{aligned}\Delta Z_1 &= P_{1x}dX - P_{1y}dY \\ &= (P_{1x} - P_x)dX + P_x dX - P_{1y}dY \\ &= (P_{1x} - P_x)dX + P_y dY - P_{1y}dY \\ &= (P_{1x} - P_x)dX + (P_y - P_{1y})dY > 0.\end{aligned}$$

Similarly the gain in system 2 from receiving dY and giving up dX is

$$\Delta Z_2 = P_{2y}dY - P_{2x}dX = (P_{2y} - P_y)dY + (P_x - P_{2x})dX > 0.$$

and the two benefits sum to the total Z benefit:

$$\Delta Z = \Delta Z_1 + \Delta Z_2 = (P_{1x} - P_{2x})dX + (P_{2y} - P_{1y})dY,$$

where $(P_{1x} - P_{2x})dX > 0$ and $(P_{2y} - P_{1y})dY > 0$. Note that both transfers now contribute to the total benefit evaluated using a numeraire not involved in the transfers.

There is no simple multiplicative conversion of ΔZ into ΔX or ΔY, because the Z must be converted into X or Y at the different rates internal to system 1 or system 2.[39] For instance, $P_{1x}dX$ would be divided by P_{1y} to get the equivalent values in Y in system 1, while $P_{2x}dX$ would be divided by P_{2y} to get the equivalent Y value in system 2. Thus, to arrive at ΔY, we would have to divide the different terms in the expression for ΔZ by the appropriate Y prices for each system:

$$\left(\frac{P_{1x}}{P_{1y}} - \frac{P_{2x}}{P_{2y}}\right) dX + \left(\frac{P_{2y}}{P_{2y}} - \frac{P_{1y}}{P_{1y}}\right) dY = (P_1 - P_2)\, dX + (1 - 1)\, dY = \Delta Y.$$

Note how the numeraire illusion appears in the mathematics as the zeroing out of the dY coefficient, that is, 1 – 1, in the calculation of ΔY (the increase in the Y pie from the transfers using Y as numeraire). In a similar manner we could convert ΔZ into ΔX, and the numeraire illusion would appear in the zeroing out of the dX coefficient in ΔX. When the numeraire illusion is avoided by evaluating the dX and dY changes in terms of some other *noninvolved* commodity Z, then we saw that both transfers added value.

With a noninvolved commodity as numeraire, the MPKH reasoning gets no illusionary foothold to recommend either dX or dY by itself on efficiency grounds.

At the cost of some complication, other commodities can be added to make the changes or project more complex. For instance, there might be other transfers dX^* from 2 to 1 and dY^* from 1 to 2 so all the transfers together were a Pareto improvement. Then with Y as the numeraire, the MPKH reasoning would recommend the dX, dX^*, and dY^* transfers on efficiency grounds. With X as the numeraire, the MPKH reasoning would recommend the dX^*, dY, and dY^* transfers on efficiency grounds.

The general result, which models a legal change or project as a set of multi-commodity and multiperson transfers (i.e., project cum compensation), is that when all the transfers together constitute a Pareto improvement, then the MPKH reasoning will recommend all the transfers except the numeraire transfers on efficiency grounds. Change the numeraire to one of the other commodities, and then the MPKH reasoning will recommend on efficiency grounds all the transfers *including* those in the old numeraire except the transfers in the new numeraire.

[39] This emphasizes the point that "changing numeraires" does not mean the trivial conversion of net benefits in one numeraire to another at some fixed public price ratio, but resummation of the benefits and costs when converted at each person's internal rate of substitution to the new numeraire.

When only two goods are involved in the transfers (as before), then the general numeraire-illusion flaw in the MPKH reasoning is illustrated in the simplest and most dramatic way as a *reversal* in the efficiency recommendations arising from a mere change in numeraire. The KH argument that efficiency does not require the numeraire transfers is only numeraire illusion. Changes in a yardstick cannot be revealed by the same yardstick – but can be revealed by changing the yardstick.

7 Justice, Mercy, and Efficiency

SARAH HOLTMAN

If one proposes to consider mercy and efficiency under the same heading, the aim surely must be to draw a contrast. For we associate mercy not only with leniency but with a fine sensitivity to circumstances and both the ability and the disposition to sympathize. No matter what the context in which we contemplate it, efficiency carries none of these associations. It requires no well-trained sensitivities or dispositions. These are time-consuming to develop and costly to employ. A mathematical formula, ready-to-hand and relatively simple to apply, much better suits efficiency's focus on savings.

This general division between mercy and efficiency carries over to their more specialized application in legal contexts. Here mercy urges attention to facts and circumstances that we might ignore if we focused solely on what strict justice requires or permits. It is, we might say, a virtuous disposition to leniency marked by a compassionate attention to the circumstances at hand.[1] Efficiency, in its relatively recent incarnation as the guiding principle of the law-and-economics approach to legal interpretation, is perhaps best understood as a means of wealth maximization. Depending on the context, we can use standards including Pareto superiority, the Kaldor–Hicks test, and the Coase theorem to determine what legal standards, or interpretations, will yield the most substantial gains. The chief concern of the economic approach is to achieve *Pareto-optimal* outcomes, those in which no distributional change could increase utility for one party without decreasing it for another. The measure of utility, of course, is individual preference, judged by parties' willingness to exchange positions or bundles of goods.[2]

[1] For a helpful overview of the features traditionally thought to characterize mercy, see Jeffrie Murphy's discussion in Jeffrie Murphy and Jean Hampton, 1988, *Forgiveness and Mercy*, New York: Cambridge University Press, p. 66.

[2] See, for example, Ronald Coase, 1960, "The Problem of Social Cost," *Journal of Law and Economics*, 3, pp. 1–44; Guido Calabresi, 1961, "Some Thoughts on Risk Distribution and the Law of Torts," *Yale Law Journal*, 70, pp. 499–553; and Richard Posner, 1973, *Economic Analysis of Law*, Boston: Little Brown. I should note that some distinctions, important in other contexts, are not significant

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