How to Understand Quantum Mechanics:

The case for the literal interpretation in terms of objective indefiniteness

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Abstract

This paper tries to elucidate the paradoxical aspects of quantum mechanics (QM) by using a simplified pedagogical model of QM based on the support sets of the state vectors, by assuming an ontology of superposition-as-objective indefiniteness, and by not giving any ontological interpretation to the computational device of the wave function. The resulting realistic interpretation is the literal or objective indefiniteness interpretation. It has already been promoted by many quantum physicists using various terminologies, and it is assumed, at least implicitly, by any non-philosophical working quantum physicist who acknowledges that a particle in a superposition state for an observable does not have a definite value for that observable prior to a measurement.

Keywords: superposition; indefiniteness; support sets; partitions; two-slit experiment

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Introduction: The literal interpretation of quantum mechanics

The problem of a realistic interpretation of quantum mechanics (QM) has been an open scandal for a century. New so-called "interpretations" are created fairly often and none, no matter how bizarre, are definitively thrown away. The conventional circular conversation in the philosophy of quantum mechanics [e.g., (Norsen 2017); (Maudlin 2019)] typically considers the Copenhagen interpretation associated with Niels Bohr (Faye 2024), together with the realistic or ontic interpretations of Bohmian mechanics (Dürr, Goldstein, and Zangi 2013), spontaneous localisation (Ghirardi, Rimini, and Weber 1986), or many-worlds (Wallace 2012).

There is another realistic understanding of quantum mechanics that Abner Shimony called the "Literal Interpretation" (Shimony 1999) and that is the approach investigated in a new way here.

> From these two basic ideas alone—indefiniteness and the superposition principle—it should be clear already that quantum mechanics conflicts sharply with common sense. If the quantum state of a system is a complete description of the system, then a quantity that has an indefinite value in that quantum state is objectively indefinite; its value is not merely unknown by the scientist who seeks to describe the system. (Shimony 1988: 47)

The key non-classical notion in QM is the notion of *superposition* which leads to states being called "inherent indefiniteness" (Feyerabend 1983), "value indefiniteness" (Stairs 1983), "incompletely determined" (Mittelstaedt 1998), "ontic vagueness" (Lowe 1994), "quantum vagueness" (French and Krause 2003), "blurry" (Rohrlich 1986), or elsewhere as unsharp, smudged, blunt, fuzzy, blob-like, dispersed, smeared out, indeterminate, or spread out. This aspect of quantum states is widely recognised as the principal problem in interpreting QM. "The central interpretative question of quantum theory is what we are to make of this [superposition] state. (Maudlin 2003: 464). Even taking into account the extension to quantum field theory, the same problem remains: "superposition, with the attendant riddles of entanglement and reduction, remains the central and generic interpretative problem of quantum theory." (Cushing 1988: 27).

How to interpret superposition?

With so much clarity about what is the central non-classical concept and main interpretive problem in QM, why has a century passed without a satisfactory interpretation of superposition emerged?

To answer that question, we must consider the mathematical formulation of QM in Hilbert spaces over the complex numbers. We don't have to really go into any details other than to note that the complex numbers are the natural mathematics to describe waves (i.e., the polar representation of a complex number has an amplitude and a phase). Hence, it is perfectly natural to think of the ontology of quantum states in terms of waves, the famous "wave function"—as if the complex numbers were needed in the mathematics of QM in order to describe an ontology of waves. But that is "not necessarily so." The field of complex numbers is the algebraically complete field extending the real numbers and that is the property needed so that the operators (observables) of QM will have a complete set of eigenvectors [see (Aaronson 2013: 119); (Weinberg 2015: 67)].

If it is thought that the complex numbers were needed to describe an ontology of waves, then the interpretation of superposition is clear; it is like the addition of waves in classical physics (e.g., electromagnetic, sound, or water waves) where the addition (i.e., superposition) of definite waves is another definite wave as in Figure 1.

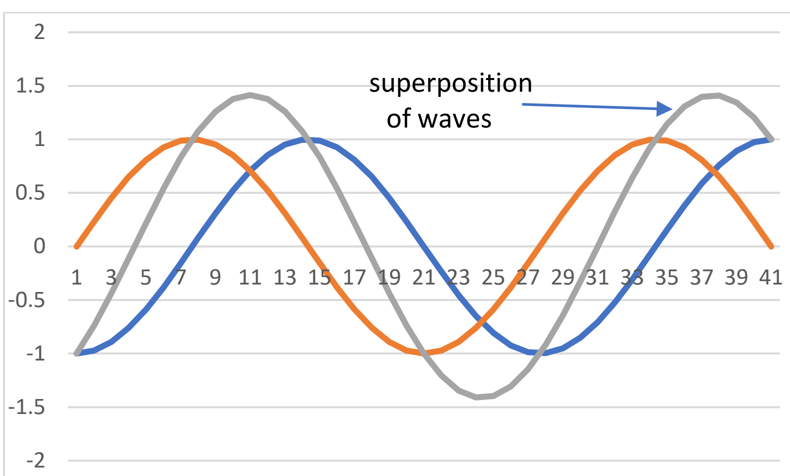

Figure 1: Superposition as addition of definite waves to give a definite wave.

When two classical (e.g., water) waves are added together, the resulting superposition wave is just as definite as the summand waves; there is no hint of indefiniteness. The new approach to superposition taken here is to reject this *classical* version of superposition-as-wave-addition in QM. Instead, the quantum superposition of two or more definite- or eigen-states should be interpreted as the state that is *indefinite or indistinct (in varying amplitudes) between the states*. This is illustrated in the real plane in Figure 2 as the addition of the definite states (1,0) and (0,1) (with equal amplitudes) to get a blurred or indefinite superposition (1,1).

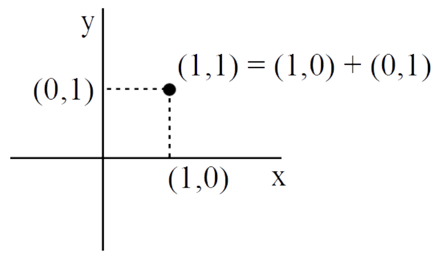

Figure 2: Superposition of definite- or eigen-states is indefinite between them

The mathematics is the same; it is the ontic interpretation that is different. To reinterpret the mathematics, we first deal with a simplified version of the mathematics.

The simplified version using sets

The German word "eigen" should be read as "definite." In a common notation for QM, a definite- or eigen-state is an example of a "ket" and is denoted $|u_i\rangle$ for i = 1,…,n, which represents n possible definite states of a quantum particle. A superposition quantum state vector is just a sum of them with complex numbers as coefficients α_i:

$$\alpha_1|u_1\rangle + \cdots + \alpha_n|u_n\rangle.$$

The *support set* is the set of the $|u_i\rangle$ with non-zero coefficients which we could write without the special ket symbols as just the set of the u_i with $\alpha_i \neq 0$. Working with just support sets yields a pedagogical model of QM called *quantum mechanics over sets*, QM/Sets (Ellerman 2024b).

Consider a simple case where there are three eigenstates $|a\rangle$, $|b\rangle$, and $|c\rangle$. In QM, there are two types of states, "pure states" and "mixed states." A *pure* state involving all the eigenstates would be a superposition of the form $\alpha|a\rangle + \beta|b\rangle + \chi|c\rangle$ with the support set $\{a, b, c\}$ if all the coefficients were non-zero. But there is also the notion of a *mixed* state where the system is in the state $|a\rangle$ with probability p_a, in state $|b\rangle$ with probability p_b, and in the state $|c\rangle$ with probability p_c (where the probabilities are positive and sum to 1). To distinguish the sets associated with these two quantum states, we need the notion of a partition (Ellerman 2024a).

Using partitions to model distinctness and indistinctness

A *partition* π on a set $U = \{u_1, \ldots, u_n\}$ is a set $\pi = \{B_1, \ldots, B_m\}$ of non-empty subsets B_j of U for j = 1,…,m, called "blocks," where the blocks are all disjoint (no overlap) and their union is all of U. For instance, $\pi = \{\{a, c\}, \{b\}\}$ is a partition of $U = \{a, b, c\}$ with {a,c} as one block and {b} as another block where the blocks have no overlap and include all the elements of $U = \{a, b, c\}$. The partitions on U can be partially ordered by saying one partition π is higher than another partition σ if the blocks of π can be obtained by breaking up some of the blocks of σ. With that ordering, the partitions on $U = \{a, b, c\}$ can be ordered in a lattice diagram as in Figure 3.[1]

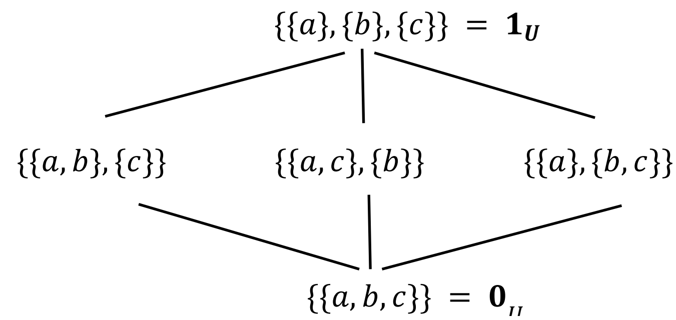

Figure 3: The lattice ordering of the partitions on $U = \{a, b, c\}$

A partition on U models which pairs of elements of the set are *indistinct* or *equivalent* to each other by being in the *same* block and which elements are *distinct* or *inequivalent* from each other by being in *different* blocks—all according to that partition. The top of the ordering is the *discrete partition* $\{\{a\}, \{b\}, \{c\}\} = \mathbf{1}_U$ where all the elements of U are distinct or distinguished from each other since all the blocks are singletons. The bottom of the ordering is the *indiscrete partition* $\{\{a, b, c\}\} = \mathbf{0}_U$ where all the elements of U are indistinct or equivalent to each other

[1] Those partitions on U represent all the supports sets that can be obtained in full QM by (projective) measurements by an observable (with the eigenvector basis U) on a quantum state with support set U.

since they are all in the same block which is U. Moving up in the ordering means having more distinctions, i.e., π is higher than σ if π makes all the distinctions of σ and more.

Now we have the mathematical machinery to model the difference between the set versions of the pure state superposition $\alpha|a\rangle + \beta|b\rangle + \chi|c\rangle$ and the completely mixed state where the system is in the state $|a\rangle$ with probability p_a, in state $|b\rangle$ with probability p_b and in the state $|c\rangle$ with probability p_c. The partition versions of those states are the indiscrete partition $\{\{a, b, c\}\} = \mathbf{0}_U$ as the support set for the pure state superposition and the discrete partition $\{\{a\}, \{b\}, \{c\}\} = \mathbf{1}_U$ as the partition version of the completely mixed state.[2] The other partitions on U are mixtures of superpositions states and mixed states. For instance, the partition $\{\{a, c\}, \{b\}\}$ is a mixture of the superposition state $\{a, c\}$ and the singleton state $\{b\}$ so b is distinguished from a and c, but a and c are not distinguished from each other by that partition.

The basic idea is to model the new interpretation of superposition at the simple level of support sets—superposition not as the addition of definite waves to give a definite wave but superposition as the state that is indistinct or blurred on where the states differ. The rendering indefinite is represented as the cohering together of the elements in a block of a partition as a class of indistinct or equivalent elements—an equivalence class. When a state breaks out of a superposition, say $\{a, c\}$ to $\{a\}$ in a "state reduction," then the state is distinguished from the superposition and becomes definite. For a more visual example of superposition, let $U = \{♣, ♦, ♥, ♠\}$, the set of suites in a deck of cards. Then the superposition state $\{♦, ♥\}$ would be indefinite on the difference between diamonds and hearts but definite on the commonality of being a red suite.

[2] Math Note: State vectors in QM can be represented by density matrices where the non-zero off-diagonal elements indicate that the corresponding elements are indistinct, i.e., cohered together in superposition. In the density matrix for a pure state, the off-diagonal entries are *indistinction (or cohering-together) amplitudes* in the sense that their (absolute) square is the probability that two independent (maximal) measurements of the state yield that pair of elements in the support set. Since superposition is the key non-classical aspect of QM, it can also be said that "the off-diagonal terms of a density matrix … are often called *quantum coherences* because they are responsible for the interference effects typical of quantum mechanics that are absent in classical dynamics. (Auletta, Fortunato, and Parisi 2009: 177) The completely decomposed mixed state has no superposition, so its density matrix is diagonal with no non-zero off-diagonal elements, i.e., no cohering together in a superposition.

Classical versus quantum physics

The indefiniteness of a superposition state is the characteristic non-classical notion in quantum mechanics. By contrast, classical physics has no objective indefiniteness—only the subjective or epistemic indefiniteness of not knowing the definite value of a quantity. Classical physics is definite "all the way down." This was captured in Leibniz's Principle of Identity of Indistinguishables (PII) (Leibniz 2000: fourth letter) and in Kant's Principle of Complete Determination.

> Every thing, however, as to its possibility, further stands under the principle of thoroughgoing determination; according to which, among all possible predicates of things, insofar as they are compared with their opposites, one must apply to it. (Kant 1998: B600)

In contrast, a superposition in QM is not completely determined since the eigenstates in the superposition are blurred or blobbed together like $\{a, c\}$ in terms of (support) sets. But when *all* the elements of the universe set U are distinguished, then that is the set version of being classical.[3] That is true only for the discrete partition.

Partition version of the PII
If u_i and u_k are indistinguishable by the discrete partition $\mathbf{1}_U$, then they are identical, i.e., $u_i = u_k$.

[3] In full QM, that discrete partition is the support set of the maximally decomposed mixed state, like the "statistical mixture describing the state of a classical dice before the outcome of the throw" (Auletta, Fortunato, and Parisi 2009: 176).

This implies that we can separate the classical and the quantum parts in the lattice ordering of partitions in Figure 3. Those two parts are like an iceberg (Kastner 2015: 1) where the visible part is classical and the underwater part is quantum (since each partition, aside from $\mathbf{1}_U$, contains at least one superposition block) as in Figure 4.

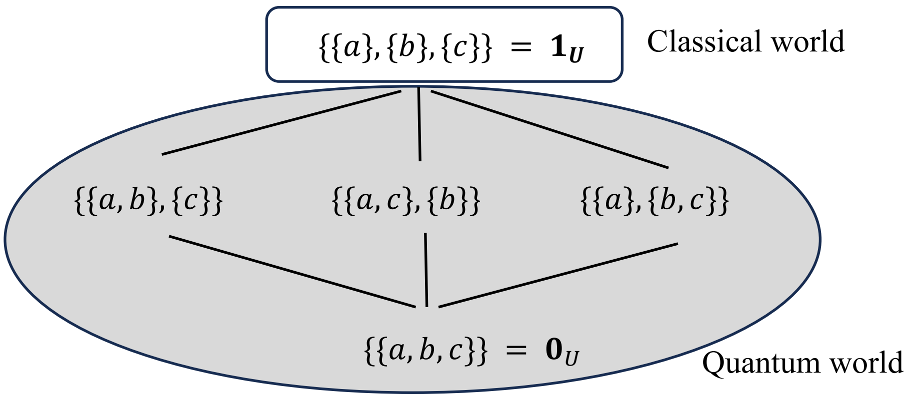

Figure 4: The "iceberg picture" of the classical and quantum worlds in a universe with three states for a quantum particle

This highly simplified picture illustrates the ontological assumption in the literal (or objective indefiniteness) interpretation of QM—the assumption that there is this underworld of particles (always quantum particles, not classical ones) in indefinite states. This is not a new view although it is usually expressed in a different vocabulary of actual states ("above the water") and potential states ("below the water").

> Heisenberg (1962: 53) used the term "potentiality" to characterize a property which is objectively indefinite, whose value when actualized is a matter of objective chance, and which is assigned a definite probability by an algorithm presupposing a definite mathematical structure of states and properties. Potentiality is a modality that is somehow intermediate between actuality and mere logical possibility. That properties can have this modality, and that states of physical systems are characterized partially by the potentialities they determine and not just by the catalogue of properties to which they assign definite values, are profound discoveries about the world, rather than about human knowledge. … These statements, together with the theses about potentiality, may collectively be called "the Literal Interpretation" of quantum mechanics. This is the interpretation resulting from taking the formalism of

> quantum mechanics literally, as giving a representation of physical properties themselves, rather than of human knowledge of them, and by taking this representation to be complete. (Shimony 1999: 6-7)

Besides Heisenberg and Shimony, many others have used this (Aristotelian) vocabulary of the two worlds of actuality (classical world) and potentiality (quantum world), e.g., (Malin 2012), (Fleming 1992), (Jaeger 2017), (Kastner, Kauffman, and Epperson 2018), (Aerts 2010), (Chiatti 2022), (deRonde 2018), (Kožnjak 2020), (Karakostas 2007), (Del Santo and Gisin 2023), and (Strumia 2021) among others. Some prefer the terminology of "latency" versus actuality instead of potentiality versus actuality, e.g., (Margenau 1954) and (Hughes 1989), and still others want to give a quantum-ontological interpretation to dispositions and propensities (see Suárez 2007 and the references therein). A better formulation would just be indefinite *actuality* versus definite actuality. That avoids the problem of accounting for a change in an ill-defined modality of "potentiality" to actuality and only requires the making-distinctions explanation about how an indefinite actuality becomes a more definite actuality—and making distinctions is precisely the way that a partition moves to a more refined partition, i.e., moves upward in the lattice diagrams. It is important to note that this is a matter of ontology, not just a manner of speaking as when one says that thrown dice have a certain potentiality, propensity, or disposition to come up with "snake eyes."

State reduction

A state reduction refers to the process by which a superposition state is reduced or "jumps" to a more definite state, if not a fully definite eigenstate. We avoid the term "measurement" since it has anthropomorphic connotations that are not appropriate for a process that takes place in the quantum world or in the transition to the classical world. The essential nature of state reduction can be seen in our simplified model; it is the transition from a non-singleton block representing a superposition state to a smaller or singleton block through *the making of distinctions or distinguishings* between the superposed states. The elements in a block representing a superposition do not represent different particles in definite states but one particle in an indefinite superposition of definite states.

A good visual representation for this process uses a grating or sieve (Weyl 1949: 255). Imagine a piece of pasta dough that has to pass through the holes in a grating. There are two possibilities. In one case, suppose the grating has each hole of a different shape and think of the ball of pasta dough as the "superposition" of the different eigen-shapes. Then as the dough passes through one of the eigen-shaped holes, it has a "state reduction" to that eigen-shape. The interaction with the grating distinguished between the superposed eigen-shapes. As Weyl put it: "Measurement means application of a sieve or grating" (Weyl 1949: 259) The second case is a "null grating" where all the holes have the same shape as the ball of dough so the passing through the grating does not distinguish between the different superposed shapes in the ball.

These two cases correspond to the two cases in the Feynman Rule (Jaeger 2014: 110) first enunciated in the early 1950s (Feynman 1951).

> When an event can occur in several alternative ways, the probability amplitude, this 'a' number [amplitude], is the sum of the 'a's for each of the various alternatives. If an experiment is performed which is capable of determining which alternative is taken, the probability of the event is changed; it is then the sum of the probabilities for each alternative. (Feynman 1967: 144-5)

This analysis contains Feynman's answer to what constitutes a state reduction (or "measurement") and what doesn't, i.e., a distinction or distinguishing between superposed states or not. The two cases are illustrated with the gratings of Figure 5.

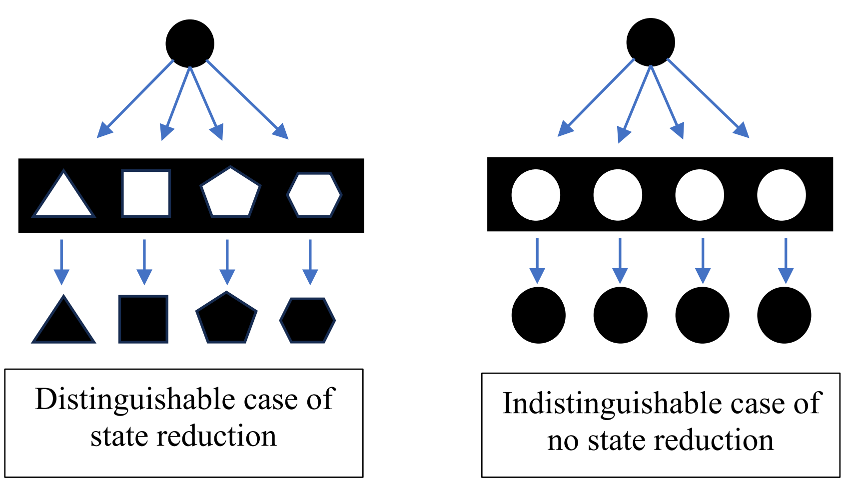

Figure 5: State reduction or not depending on whether the interaction makes distinctions

Also in our simplified model, it is distinctions that split up a superposition into a more definite, if not fully definite eigenstate. For instance, in the mixed state $\{\{a,c\},\{b\}\}$, the distinction between a and c in $\{a,c\}$ would give the fully definite state $\{\{a\},\{b\},\{c\}\}$.

How are distinctions made in QM? They are made by the interaction (like the passing of the pasta ball through the grating in the distinguishable case) that applies the distinctions of an observable (the four eigen-shapes in Figure 5) to a quantum state (the pasta ball as a particle in the superposition of the four eigen-shapes). The distinctions do not have to be observed or even be observable by humans, but "Nature knows."

The measurement operation is described by the Lüder's mixture operation (Lüders 1951) which transforms a superposition state into a mixture of states one of which occurs with a certain probability. In the simplified QM/Sets model, that operation is just the join of two partitions (Ellerman 2024b: 480-1), one representing the observable and the other representing the state being measured, a pure state like $\{\{a,b,c\}\}$ or a mixed state like $\{\{a,c\},\{b\}\}$. The *join* of two partitions is the new partition whose blocks are the non-empty intersections of two blocks, one from each partition. Suppose that $\{\{a,c\},\{b\}\}$ is the state being measured. And suppose that the partition $\{\{a,b\},\{c\}\}$ represents the observable being measured. That means the observable's eigenstates a and b have the same definite- or eigen-value associated with them and that c has a different eigenvalue associated with it. Then the join (represented by the $\vee$ symbol) of the two partitions is the discrete partition: $\{\{a,b\},\{c\}\}\vee\{\{a,c\},\{b\}\} = \{\{a\},\{b\},\{c\}\}$. This is represented in the lattice ordering of partitions in Figure 6.

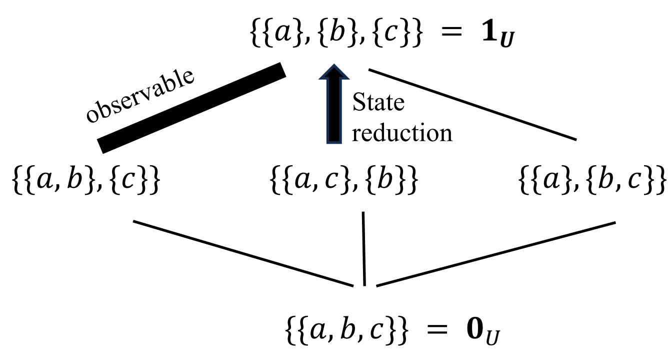

Figure 6: Join of $\{\{a,b\},\{c\}\}$ and $\{\{a,c\},\{b\}\}$ to yield $\{\{a\},\{b\},\{c\}\} = \mathbf{1}_U$

When superposition is thought of as wave-addition, then the measurement or state reduction seems like a mysterious process called the "collapse of the wave packet." The interpretation of superposition as rendering the separate eigenstates as indistinct (with various indistinction amplitudes) where they differ was not an *ad hoc* choice. It is the reverse of state reduction that (by the Feynman rules) occurs by making the superposed states distinct. Since state reduction is the *undoing* of superposition by making the superposed states distinguishable, the *formation* of a superposition must be the making of the superposed states indistinct or indistinguishable (in varying degrees). That is how the interpretation of superposition and the criterion for state reduction are related.

Superposition and state reduction as opposite processes. The making of a superposition is a process of making the states indefinite or indistinct where they differ and state reduction is the *opposite* process of making the superposed states distinguishable or distinct.

Making the superposed eigenstates distinguishable (as in the left side of Figure 5) breaks up the superposition into more definite or fully definite mixed states, one of which occurs with some probability.

A realistic example is often used by Feynman (Feynman, Leighton, and Sands 2010: §3.3) to illustrate the interactions that do make distinctions or do not make distinctions (as in Figure 5). A neutron is scattering off the nuclei of atoms in a crystal. If the nuclei have no spin, then the amplitude for the neutron to be scattered to some given point would be the superposition of the scattering amplitudes off the various nuclei since there is no distinguishing physical event to distinguish between scattering off one nucleus or another. But if all the nuclei had spin in, say, the down direction while the neutron had spin up, then in the scattering interaction, one of the nuclei might flip its spin which would be the physical event to distinguish that trajectory. Then the probability of the neutron arriving at the given point with its spin reversed (indicating that a spin flip had occurred) would be the sum of the probabilities (not the amplitudes) for those distinguished trajectories over all the nuclei. In that case, the superposition was reduced (the indefinite became definite) and the nucleus with its spin flipped plays the role of a detector registering that the scattering was off that nucleus. The spin-state of the nuclei served as a measuring apparatus to measure which scattering trajectory was taken by the neutron to reach the detector.[4]

State reduction is where the probabilistic indeterminacy enters QM. In the example of Figure 4, the superposition of the four eigen-shapes is reduced to one of those eigen-shapes with some probability. That probability is not based on the subjective lack of knowledge about all the detailed causes—as it would be for throwing a die. It is an objective probability. In terms of the support sets, state reduction is the transition from a non-empty set of elements to one of the elements.

Given a whole set of non-empty sets, a function that picks out an element of each set is called a *choice function* (Halmos 1974: 60). With no structures on the non-empty sets, there is no way that the choice is determinate—which is why the Axiom of Choice (i.e., that choice functions always exist) has to be added as an independent axiom to axiomatic set theory. That indeterminate choice of an element of each set is the set version of the objective indeterminacy of state reduction in QM. But there is one case where the choice function is determined and that is when each non-empty set is a singleton. And, accordingly, that is also the case in QM when the result of a measurement or state reduction is completely determined, namely when the state being measured is not a superposition of different eigenstates but is an already a single eigenstate (in the observable being measured)—so the support set is a singleton—as illustrated in Table 1.

Choice function f	**Quantum state reduction (maximal)**
$f(\{a, b, \dots, c\}) = b$	$\alpha\vert a\rangle + \beta\vert b\rangle \dots + \gamma\vert c\rangle \to \vert b\rangle$ (indeterminate)
$f(\{b\}) = b$	$\vert b\rangle \to \vert b\rangle$ (determinate)

Table 1: Indeterminate and determinate cases

The two fundamental processes of von Neumann

John von Neumann (Von Neumann 1955) famously specified that there are two fundamental processes in QM: the type I process of state reduction and the type II process of unitary evolution (according to Schrödinger equation). We have already characterised the type I processes as ones

[4] The classical world (Figure 4) should not be identified with the macroscopic human-observed world. The distinction, made by the change of spin in the nucleus due the particle scattering off of it, has nothing to do with a macroscopic measuring apparatus.

that make distinctions. Hence the natural characterisation of the type II processes would be ones that don't make distinctions. But what does that mean?

We first answer that question at the level of support sets and that answer naturally extends to full QM. We want a measure of how much two subsets $S, T \subseteq U$ with the same cardinality $|S| = |T| = k$, i.e., same number of elements, are distinct or indistinct. The natural measure of indistinctness is the cardinality of overlap or intersection $S \cap T$ of the two sets (i.e., the set of common elements). If $|S \cap T| = k$ then the two sets are completely indistinct (i.e., are the same set) and if $|S \cap T| = 0$ then the sets are totally distinct (or have zero indistinctness), i.e., are disjoint.

Our support sets of state vectors in QM use the representation of the vector in terms of a certain (basis) set of eigenstates U. But there are many different basis sets of eigenvectors, e.g., $U' = \{u'_1, \ldots, u'_n\}$ always of the same cardinality. When a quantum system evolves in time, a state evolves to a state that can be expressed in a different set of eigenvectors, e.g., so the support set goes from a subset of U to a subset of some U'. We want to single out the evolutions that preserve the degree of indistinctness (so no distinctions are made).

If S and T expressed in terms of U evolve to S' and T' expressed in terms of U', then preserving the degree of indistinctness means that $|S \cap T| = |S' \cap T'|$. In full QM, the overlap or degree of indistinctness between two (normalised) quantum states is their inner product, so an evolution that preserves the degree of indistinctness (and thus creates no distinctions) is an evolution that preserves inner products. That is a unitary transformation and that is precisely von Neumann's type II process. Hence von Neumann's two fundamental processes are those that create distinctions and those that don't. In terms of the ordering of partitions on U and on U', the type I state reductions move vertically upward (i.e., more distinctions) as in Figure 6 and the type II evolutions move horizontally (or downward) from a partition expressed in U to a partition expressed in U' as illustrated in Figure 7 where $U = \{a, b, c\}$ and $U' = \{a', b', c'\}$.

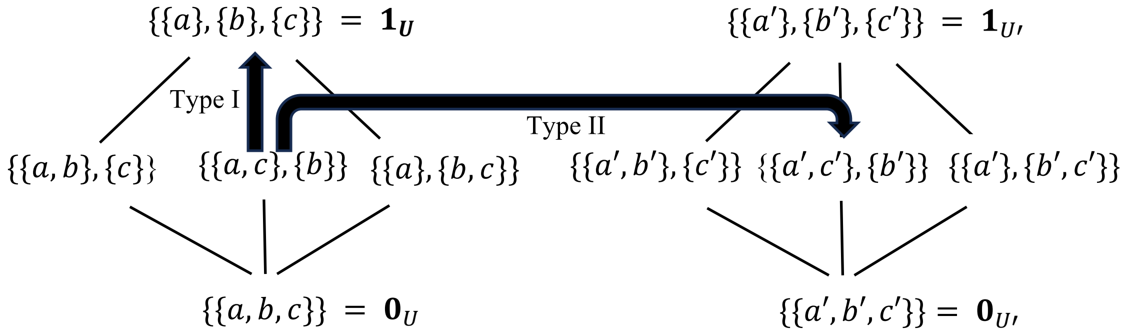

Figure 7: Type I and Type II processes

The two-slit experiment

Adding sets as vectors

The two-slit (or double-slit) experiment is Richard Feynman's favorite example of the mysteries of QM; in fact, he said it contains the "only mystery" (Feynman, Leighton, and Sands 2010: §1.1). Our expository strategy has been to develop simple intuitive QM/Sets models of quantum phenomena using support sets of quantum states rather than using the full mathematics of Hilbert spaces.

To develop a support set version of the crucial two-slit experiment, we need to see how these sets can be treated as vectors in a vector space over the field of integers modulo 2 where $1 + 1 = 2 = 0$. When this addition is applied to subsets, this means that, for instance, $\{a, c\} + \{b, c\} = \{a, b\}$ (since $2c = 0$). Then when we move from one basis set of eigenstates like $U = \{a, b, c\}$ to another set $U' = \{a', b', c'\}$, each vector has a different representation in each basis. In Table 2, each column is the same underlying vector but represented in different basis sets U and U' so, for instance, $\{b'\} = \{a, b, c\}$ and $\{c'\} = \{b, c\}$, so $\{b', c'\} = \{a, b, c\} + \{b, c\} = \{a\}$.

***U* basis**	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b, c\}$
***U′* basis**	$\{b', c'\}$	$\{a', b', c'\}$	$\{a', b'\}$	$\{a'\}$	$\{a', c'\}$	$\{c'\}$	$\{b'\}$

Table 2: Each column is same abstract vector represented in U-basis and U'-basis

The setup for the two-slit experiment

In the actual experiment, there is a screen with two narrow slits in it so that a particle fired at the screen can go through slit 1 or slit 2, and then the particle evolved to a detection wall where its impact is recorded in a tiny dot. There are two cases. In case 1, there are detectors at the slits so that it is detected which slit the particle went through. As many particles are fired at the screen, one at a time, eventually a certain pattern emerges on the detection wall. But if there are no detectors at the slits, then one might think the same pattern would emerge on the detection wall since common sense assumes that each particle still has to go through one slit or the other to get to the detection wall. But then a quite different pattern emerges of stripes on the wall as if the particle suddenly turned into a wave that went through both slits and interfered with itself (with constructive and destructive interference in different regions) to only turn into a particle again when it hits the detection wall.

Our expository strategy is to use a highly simplified QM/Sets model using the support sets of the state vectors. Although it is not an actual physical setup, the set version of the experiment captures enough of the actual experiment to reproduce the seemingly paradoxical aspects—and to offer an explanation using the concepts we have developed so far (without "wave-talk"). In this skeletal model, the three eigenstates represent vertical positions on the screen and on the wall. On the screen, the two slits are at positions $\{a\}$ and $\{c\}$ as illustrated in Figure 8.

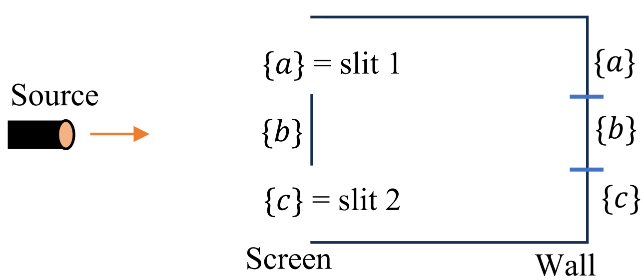

Figure 8: Setup for two-slit experiment

In one time unit, the particle goes from the source to the screen and then in another time unit from the screen to the wall by the dynamics:

$$\{a\} \to \{a'\} = \{a, b\}; \{b\} \to \{b'\} = \{a, b, c\}; \{c\} \to \{c'\} = \{b, c\}.$$

The particle is emitted from the source at $\{b\}$ and spreads out to $\{b'\} = \{a, b, c\}$ by the time it arrives at the screen. If the particle hits the screen at $\{b\}$ then the superposition reduces to $\{b\}$. Otherwise, the particle's state is reduced to $\{a, c\}$ so that superposition state is how the particle arrives at the slits. Thus, the interaction with the screen reduces the pure state partition $\{\{a, b, c\}\}$ to the mixed state $\{\{a, c\}, \{b\}\}$ and we are concerned with the evolution of the state $\{a, c\}$ which is essentially the superposition $\{slit\ 1, slit\ 2\}$.

Case 1: Detection at the slits

If there are detectors at the slits, then that interaction distinguishes between the eigenstates in the superposition, so it is reduced to the state $\{a\}$, i.e., going through slit 1, or to the state $\{c\}$, i.e., going through slit 2, with half-half probability. In the slit 1 case, the particle evolves to the state $\{a'\} = \{a, b\}$ and that superposition reduces to either $\{a\}$ or $\{b\}$ at the detection wall with half-half probability. In the slit 2 case, the particle in state $\{c\}$ evolves to the state $\{c'\} = \{b, c\}$ which reduces at the wall to $\{b\}$ or $\{c\}$ with half-half probabilities resulting in the probability distribution in Figure 9.

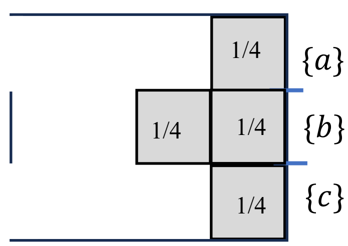

Figure 9: Case 1 hit probabilities with detectors at slits

Thus, the probability of the particle hitting at $\{a\}$ is $\frac{1}{2}\frac{1}{2} = \frac{1}{4}$ and similarly for $\{c\}$. But there are two distinguished ways the particle could hit at $\{b\}$ so it has probability $2\left(\frac{1}{2}\frac{1}{2}\right) = \frac{1}{2}$.

Case 2: No detection at the slits

If there are no detector at the slits, then there is no distinguishing between the elements in the superposition $\{a, c\}$ at the screen so it evolves as:

$$\{a, c\} = \{a\} + \{c\} \rightarrow \{a'\} + \{c'\} = \{a. b\} + \{b, c\} = \{a, c\}.$$

With the destructive cancellation of $\{b\}$, the superposition $\{a, c\}$ hits the detection wall and reduces to $\{a\}$ or $\{c\}$ with half-half probability. Then the bar graph of the hit probabilities shows the stripes characteristic of an interference pattern as in Figure 10.

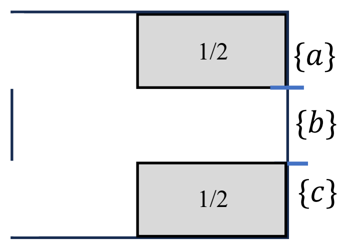

Figure 10: Case 2 hit probabilities with no detection at slits showing characteristic stripes due to destructive interference at $\{b\}$

Analysing the paradoxical aspects of the two-slit experiment

The first point to be made is that in Case 1, the essential point is that the detectors distinguished between the superposed states. There is no need for humans to observe the distinction.

> In other words, the superposition of amplitudes in [Case 2] is only valid if there is no way to know, even in principle, which path the particle took. It is important to realize that this does not imply that an observer actually takes note of what happens. It is sufficient to destroy the interference pattern, if the path information is accessible in principle from the experiment or even if it is dispersed in the environment and beyond any technical possibility to be recovered, but in principle still "out there." The absence of any such information is the essential criterion for quantum interference to appear. (Zeilinger 1999: 484)

In QM, there are no classical *spatial trajectories* of a particle between two points in space. In Case 1, the detectors localise the particle at the classical point $\{a\}$ (slit1) or at $\{c\}$ (slit 2). But then further evolution from $\{a\}$ to $\{a'\} = \{a, b\}$ takes place at the quantum level, not in the space of distinguished points in the classical world, and similarly for the evolution of $\{c\}$ to $\{c'\} = \{b, c\}$. Then in those two subcases, the detection wall distinguishes $\{a. b\}$ into $\{a\}$ or $\{b\}$, and similarly, the detection wall distinguishes $\{b, c\}$ into $\{b\}$ or $\{c\}$—as illustrated in Figure 11.

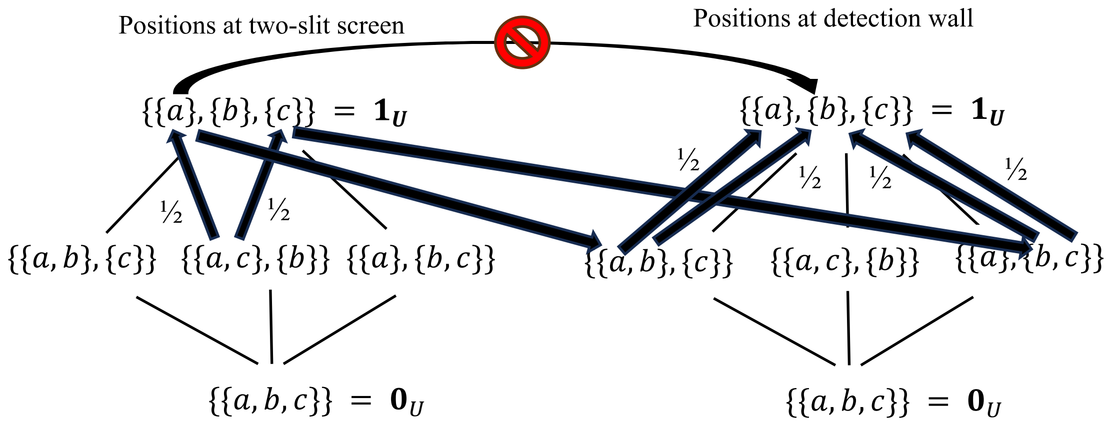

Figure 11: Case 1 evolutions with no spatial (i.e., classical) trajectory from a classical state at the screen to a classical state at the wall

As David Hawkins put it, the picture "is no longer that of a steady deterministic flow..., but that of states and transitions, 'flights and perchings,' in which the perchings are more stable and flights more abrupt than classical ideas would have allowed." (Hawkins 1964: 198) Getting from a distinguished (i.e., classical) state at the screen (1^{st} perch) to a distinguished state at the wall (2^{nd} perch) involves evolution (flight) only in the quantum world. Each of the upward arrows represents a state reduction (or a type I process of making distinctions) and the other horizontal or downward arrows represent non-probabilistic evolution (or a type II process not making distinctions). In terms of superpositions, the upward arrows visually represent the unraveling or undoing of a superposition while the non-upward (i.e., downward or horizontal) arrows represent the formation of superpositions. Seeing those two processes as opposites provides the logic behind interpreting superposition as the rendering indistinct of the superposed states as opposed to state reduction which distinguishes them in a mixture of eigenstates, one of which occurs with a certain probability.

Probabilities are introduced only in the state reductions. Under the simplest assumptions working with just sets, the probabilities of a binary superposition reduction to either eigenstate is half-half. Hence a 1/2 probability can be assigned to each upward arrow and, since the paths are in principle distinguishable, the probabilities along each path multiply (Jaeger 2014: 111, Feynman rule 3.5) giving the bar chart of probabilities in Figure 8 for Case 1.

When there are no detectors in Case 2, then "common sense" tells us that the particle still has to go through one slit or the other to get to the detection wall. But we saw in Case 1 that if it *definitely* goes through one of the slits, then there is no striped interference pattern—so in Case 2, how does the particle get to the detection wall? That is one way to pose the paradoxical aspect of QM revealed by the two-slit experiment. The "answer," that seems to help many quantum theorists and physics teachers sleep better, is to suddenly switch to wave-talk (i.e., the magic called "wave-particle complementarity") and to say that the wave goes through both slits, like the water waves in the classroom ripple-tank demonstration, that the two parts then interfere with each other, and that "accounts" for the striped interference pattern at the wall.

The analysis in terms of superposition-as-indistinction gives a different *ontological* account—since the mathematics is the same regardless of the ontic interpretation of superposition. The two events, the particle going through slit 1 and the particle going through slit 2, are the events $\{a\}$ or $\{c\}$ in the classical world represented in the discrete partition $\{\{a\},\{b\},\{c\}\}$ giving the three possible classical (i.e., distinguished) events. But in Case 2, there was no distinction made at the screen so the superposition $\{a,c\}$ was not broken up; it does not emerge to the classical level. Thus, it remained *at the non-classical level* of the quantum world of indefiniteness and then evolved *at that level* to $\{a',c'\} = \{a,c\}$ as shown in Figure 12.

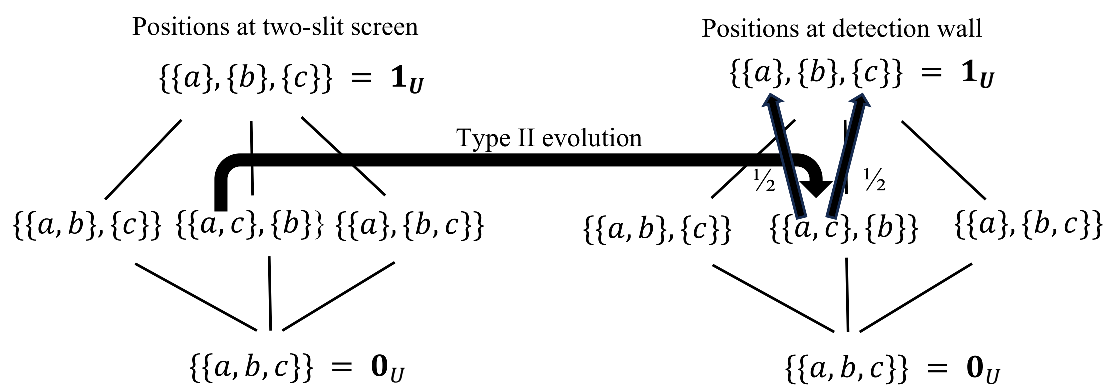

Figure 12: Case 2 evolution at quantum level followed by distinctions at the detection wall

Here again, probabilities are introduced only for state reductions (upward arrows), and they are half-half for the only state reduction at the detection wall giving the bar graph of probabilities for Case 2 in Figure 10.

Our intuitions from the macroscopic physical world are not much help in "seeing" how this evolution in the indefinite quantum underworld gets the superposition to the detection wall without the particle classically going through one slit or the other. Classical states “emerge” from indefinite states by the making of distinctions. A metaphor for definiteness emerging from indefiniteness is a police artist’s facial sketch pad where an indefinite outline slowly changes to become a more realistic definite face. Another metaphor might be 3D printing where each layer—placed here rather than there—slowly creates a distinctive object. An even better visualization is provided by the generative AI programs that develop an image based on a written description. The generated image starts from an indefinite blurred picture and then is refined more and more through each iterative jump to reach the final definite image. That is “essentially” how the classical world emerges from the quantum world.

The physics Nobel laureate, Anthony Leggett, when making a presentation to physics departments, would take a poll of how many agreed with the (negative) statement about Case 2: “the particle does not go through slit 1 or slit 2,” and he gladly reported “I almost invariably get a large majority in favor.” (Leggett 2011: 155) One wonders how many agreed because they rejected the particle-talk in favor of wave-talk in that case.

Feynman is quite clear on this point: “We must conclude that when both holes are open it is *not true* that the particle goes through one hole or the other” (Feynman 1951: 536) and he does not try to explain it by reverting to wave-talk. He does not give an ontological interpretation to the mathematical device of the wave function since “it must be emphasized that the wave function that satisfies the [Schrödinger] equation is not like a real wave in space; one cannot picture any kind of reality to this wave as one does for a sound wave.” (Feynman, Leighton, and Sands 2010: §3.7)

This analysis of the two-slit experiment can be used to elucidate other “paradoxes” created by thinking as if there is only the fully definite classical world and ignoring the quantum underworld of quantum particles in indefinite states. For instance, if the wave function was thought of as an ontological wave (like the water waves in the misleading classroom ripple-tank “demonstration” of the two-slit experiment), then the “collapse of the wave packet” in a state reduction would involve a movement of matter across a spatial distance faster than the speed of light, e.g., Einstein’s “spooky action at a distance.” But the wave function is not an ontic entity, and the spread-out wave function represents a superposition (of positions) that does not exist in the space of distinguished spatial points. In that sense, the quantum underworld of indefiniteness is non-spatial (in contrast to Einstein's notion of "distance").

The same holds in the analysis of entangled particles. Entanglement is a special type of superposition so that connection between two particles, said to be "spread out in space," is a non-spatial connection. Superpositions exist at the quantum level and only rise to the classical level with state reductions which are probabilistically indeterminate.[5]

Concluding remarks

The realistic literal or objective indefinite interpretation of QM presented here is based on two premises:

1) that the ontology underlying QM is that of objective indefiniteness or indistinctness, not that of waves, and

2) that a superposition of definite- or eigen-states should be interpreted as a state that is indistinct or indefinite between the definite eigenstates (as in Figure 2)—not that of the addition of definite waves to get a definite wave (as in Figure 1).

QM is formulated in vector spaces over the field of complex numbers since that field (unlike the real numbers) is algebraically complete and thus the observable operators will have a complete set of eigenvectors—not because the theory is describing a wave ontology. The complex numbers are also the natural mathematics to describe waves so that wave interpretation of the mathematics will always be there automatically. There is nothing particularly 'wavy' in constructive or destructive interference, since it is there for the addition of vectors in *any* vector space whatsoever (as in the QM/Sets model of the two-slit experiment). But in a vector space

[5] That indeterminacy is why entanglement does not allow communication faster than the speed of light in space. Imagine a "communication device" where Alice pressed the 0-buttom or the 1-button to send messages to Bob. But after pressing either button there was a fair coin flip ($H = 1$ and $T = 0$) to determine whether a 0 or 1 was instantly sent to Bob. No communication from Alice to Bob would be possible.

over the complex numbers, the ontic wave misinterpretation of that constructive or destructive interference is always present (Hughes 1989: 303).

The tendency to assume an ontological interpretation of the wave mathematics has been one of the causes of the failure to literally interpret QM for a century. Einstein said, "the Lord is subtle, but not malicious." While not malicious, the century-long misdirection of the wave mathematics shows that He may be, at least, a trickster.

If the ontology is not that of waves, then a new realistic interpretation must be developed for superposition other than the addition of definite waves to give an equally definite wave. Every quantum theorist who acknowledges that a superposition state does not have a definite value prior to a state reduction ("measurement") already knows that new interpretation of superposition-as-indefiniteness—and already assumes, at least implicitly, a quantum underworld of indefiniteness. The ontological cost of this approach is that assumption of the existence of that quantum underworld of indefiniteness—also labeled as "potentialities" (or "latencies") by Heisenberg, Shimony, and a host of others.

We have developed that indefiniteness interpretation here using a new expository approach of AM/Sets which deals with partitions of support sets of the state vectors of (finite-dimensional) Hilbert space (Ellerman 2024c).[6] That greatly simplifies the mathematics to relatively intuitive models that nevertheless capture and explain the paradoxical aspects of the full theory as in the two-slit experiment. Then the Literal or Objective Indefiniteness Interpretation is more easily understood.

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[6] Math Note: There is a mathematical subset-partition duality (Ellerman 2024d) that runs throughout the exact sciences. This treatment of QM is part of the partition side of the duality that goes from the mathematical logic of partitions (Ellerman 2023), to the quantitative version in logical information theory (Ellerman 2021), to the quotient-object side of the reverse-the-arrows duality of category theory (Ellerman 2022), and then to this treatment of QM (Ellerman 2024c)—which is thus a coherent part of a larger mathematical picture.

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